Answer on Question #40689, Physics, Mechanics

The acceleration of a particle is increasing linearly with time t as bt . The particle starts from origin with an initial velocity v_o. The distance travelled by the particle in time t will be?

Solution:

Given:

$a(t) = bt$

The derivative of a distance function represents instantaneous velocity and that the derivative of the velocity function represents instantaneous acceleration at a particular time.

Thus,

$$a = \frac{dv}{dt} = bt$$

Integrating, the equation

$$\int dv = \int bt\, dt$$

$$v = \frac{bt^2}{2} + C$$

where C is constant of integration.

At t = 0, $v = v_0$, hence $C = v_0$ and equation becomes

$$v = \frac{bt^2}{2} + v_0$$

We have that

$$v = \frac{dx}{dt}$$

where x is distance

$$\frac{dx}{dt} = \frac{bt^2}{2} + v_0$$

Integrating

$$\int dx = \int \left(\frac{bt^2}{2} + v_0\right) dt$$

$$x = \frac{b}{2}\frac{t^3}{3} + v_0 t + x_0$$

where x_0 is constant of integration.

At t = 0, $x = 0$, hence $x_0 = 0$ and equation becomes

$$x = \frac{bt^3}{6} + v_0 t$$

Answer. $x = \frac{bt^3}{6} + v_0 t$.