Answer on Question #40677 – Physics – Mechanics

A stone is projected from a point on the ground so as to hit a bird on the top of a vertical pole of height h and then attain a max. height 2h above the ground . If at the instant of projection the bird flies away horizontally with a uniform speed and the stone hits the bird while descending , then ratio of speed of bird to the horizontal speed of the stone is ?

Solution:

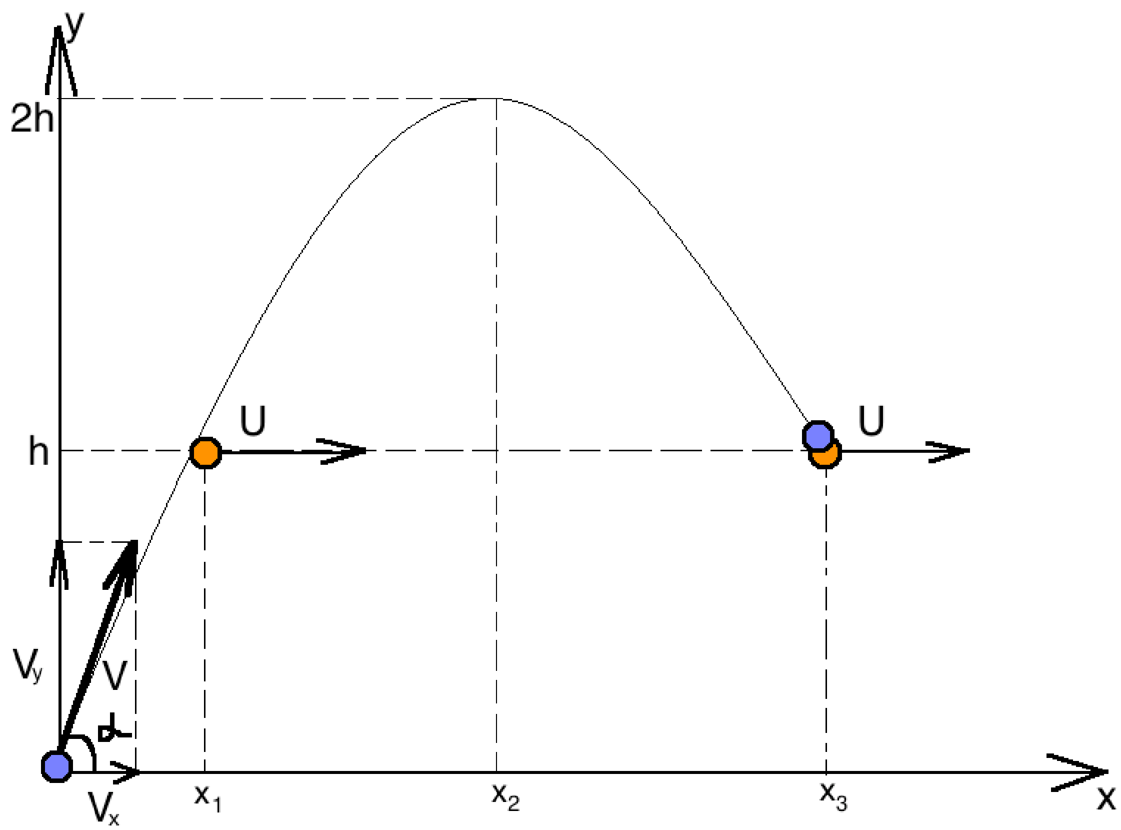
2h – maximum height of the flight;

V — initial speed of the stone;

$U -$ *horizontal speed of the bird*;

α — the angle between the projection and the horizontal;

h — height at which the bird flies;



These problems can be rather tricky with the number of equations that can be written down. If you draw the parabolic arc representing the path of the stone, this path must pass through the top of the pole (let its coordinates there be (x_1, h) with h = height of the pole) then to the top of its flight with coordinates $(x_2, 2h)$ and then to the point where it brushes the bird at coordinates (x_3, h). Then we note that if t_1, t_2, t_3 are the times for the stone to reach x_1, x_2, x_3 respectively, then t3 is also the time for the bird to travel a distance $(x_3 - x_1)$ at speed U, that is $U \cdot t_3 = x_3 - x_1$ and this is the equation I shall be using to find the horizontal velocity of the stone.

$V_x = V\cos\alpha -$ horizontal velocity of the stone;
$V_y = V\sin\alpha -$ vertical velocity of the stone;

Equation of the motion for the stone (horizontal motion):

$$x\colon x = Vt\cos\alpha$$

where α is the angle of elevation and V is the velocity of projection. For vertical motion:

$$y\colon h = Vt\sin\alpha - \frac{gt^2}{2}$$

Time to highest point is given by the equation $'v = u + at'$ which has $v = 0$ at the highest point. Thus

$$0 = V\sin\alpha - gt \Rightarrow t = \frac{V\sin\alpha}{g}$$

y = 2h at the highest point, so we can write:

$$2h = V\sin\alpha \cdot \frac{V\sin\alpha}{g} - \frac{g}{2}\left(\frac{V\sin\alpha}{g}\right)^2 = \frac{V^2\sin^2\alpha}{2g}$$

$$h = \frac{1}{4}\frac{V^2\sin^2\alpha}{g}$$

Now we can use this result to find the two times when the stone is at height h, and hence find t_1 and t_3 and x_1 and x_3.

We have:

$$h = Vt\sin\alpha - \frac{gt^2}{2}$$

$$\frac{1}{4}\frac{V^2\sin^2\alpha}{g} = Vt\sin\alpha - \frac{gt^2}{2}$$

$$\frac{gt^2}{2} - Vt\sin\alpha + \frac{1}{4}\frac{V^2\sin^2\alpha}{g} = 0$$

$$2g^2t^2 - 4gV\sin\alpha\, t + V^2\sin^2\alpha = 0$$

$$t = \frac{4gV\sin\alpha \pm \sqrt{16V^2g^2\sin^2\alpha - 8V^2g^2\sin^2\alpha}}{4g^2} = \frac{4gV\sin\alpha \pm \sqrt{8V^2g^2\sin^2\alpha}}{4g^2} =$$

$$= \frac{2gV\sin\alpha \pm 2\sqrt{2}gV\sin\alpha}{2g^2} = \frac{V\sin\alpha\left(2 \pm \sqrt{2}\right)}{2g}$$

So $t_1 = \frac{V\sin\alpha(2-\sqrt{2})}{2g}$ *and* $t_2 = \frac{V\sin\alpha(2+\sqrt{2})}{2g}$

To get $x_3 - x_1$ we multiply by $V\sin\alpha$

$$x_3 - x_1 = \frac{V^2\cos\alpha\sin\alpha\left(2+\sqrt{2}-2+\sqrt{2}\right)}{2g} = \frac{2\sqrt{2}V^2\cos\alpha\sin\alpha}{2g} = \frac{\sqrt{2}V^2\cos\alpha\sin\alpha}{g}$$

Now equate this to $U \cdot t_3 = distance\ flown\ by\ bird$:

$$\frac{\sqrt{2}\mathrm{V}^2\cos\alpha\sin\alpha}{g} = U\frac{\mathrm{V}\sin\alpha\left(2+\sqrt{2}\right)}{2g}$$

$$V\cos\alpha = U\frac{2+\sqrt{2}}{2\sqrt{2}} = \mathrm{V_x}$$

Then ratio of speed of bird to the horizontal speed of the stone is

$$\frac{U}{V_x} = \frac{2+\sqrt{2}}{2\sqrt{2}} = 1.2$$

Answer: ratio of speed of bird to the horizontal speed of the stone is 1.2