Answer on Question #40557 – Physics – Electrodynamics

Question: charge Q is uniformly distributed over a half ring of radius R. The electric field at the center of the ring is

1) $\frac{Q}{2\pi^2\epsilon R^2}$;
2) $\frac{Q}{2\pi\epsilon R^2}$.

Solution: charge $dq = \frac{Q}{\pi R} dl$ $(dl = Rd\theta)$ creates in the center of the ring the electric field

$$dE = \frac{dq}{4\pi\epsilon R^2}$$

This field has two components

$$dE_x = dE \cdot \cos\theta\,, \quad dE_y = dE \cdot \sin\theta.$$

After integrating, we obtain the value of the electric field in the center

$$E_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dE \cdot \cos\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dq}{4\pi\epsilon R^2} \cdot \cos\theta = \frac{Q}{4\pi^2\epsilon R^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta = \frac{Q}{2\pi^2\epsilon R^2};$$

$$E_y = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dE \cdot \sin\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dq}{4\pi\epsilon R^2} \cdot \sin\theta = \frac{Q}{4\pi^2\epsilon R^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\theta\, d\theta = 0;$$

Finally, the resulting value is

$$E = E_x = \frac{Q}{2\pi^2\epsilon R^2}.$$

Answer: the electric field in the center is

$$1)\ E = \frac{Q}{2\pi^2\epsilon R^2}.$$