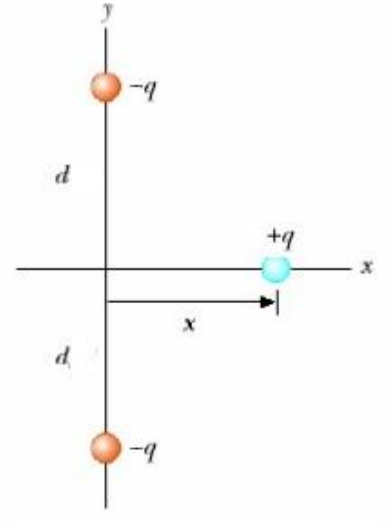
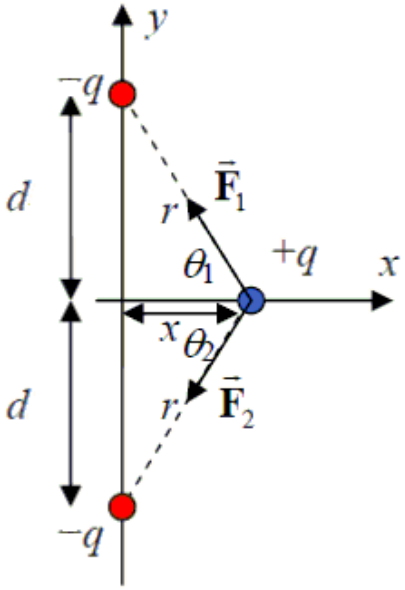
Answer on Question #40391, Physics, Electric Circuits

two charges(-q)each are separated by a distance 2d.A third charge q of mass m placed at the mid-point is displaced slightly by a distance x ($x \ll d$) perpendicular to the line joining the two fixed charges .show that charge q will perform SHM

Solution





The net force on the q charge is $\vec{F} = \overrightarrow{F_1} + \overrightarrow{F_2}$. Since $F_1 = F_2 = \frac{kq(-q)}{r^2} = -\frac{kq^2}{r^2}$, with $r = \sqrt{x^2 + d^2}$ and from the symmetry $\theta_1 = \theta_2 = \theta$ so the y–components of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ cancel. So the magnitude of $\vec{F}$ is $F = F_1 \cos\theta + F_2 \cos\theta$ with $\cos\theta = \frac{x}{r}$. Therefore

$$F_x = -F = -2F_1 \cos\theta = -2\left(\frac{kq^2}{r^2}\right)\frac{x}{r}.$$

Now,

$$ma_x = F_x \to a_x = \frac{-2kq^2x}{m\left(\sqrt{x^2 + d^2}\right)^3}.$$

If $x \ll d$

$$a_x \approx \frac{-2kq^2}{m(d)^3}x = -\frac{2kq^2}{md^3}x = -\omega^2 x.$$

This is in the form for Simple Harmonic Motion with $\omega = \sqrt{\frac{2kq^2}{md^3}}$.