Answer on Question #40207 – Physics – Physics, Mechanics | Kinematics | Dynamics

**1.** A ball of mass 1.5 kg rolling to the right with a speed of 3.6 ms^-1 collides head-on with a spring with a spring constant of 2.0 Nm^-2. Determine the maximum compression of the spring and the speed of the ball when the compression of the spring is 0.10 m.

$m = 1.5 kg$

$v_0 = 3.6 \frac{m}{s}$

$k = 2 \frac{N}{m}$

$x_1 = 0.1 m$

$x_{\max}, v_1 - ?$

*Solution.*

The initial kinetic energy of the ball can be found according to Steiner's theorem: $W = \frac{I\omega^2}{2} + mr^2$, where $I = \frac{2mr^2}{5}$ is momentum of inertia regarding to the axis across the center of the ball, $\omega = \frac{v_0}{r}$ is the initial angular speed of the ball.

So, $W = \frac{6mv_0{}^2}{5}$.

As the total mechanical energy of the system is conserved, then

$$W = \frac{kx^2}{2} + \frac{6mv^2}{5},$$

where $x$ and $x$ are the compression of the spring and the velocity of the ball at any moment.

The maximum compression of the spring can be found assuming $v = 0$:

$$\frac{6mv_0{}^2}{5} = \frac{kx_{\max}{}^2}{2}, \quad \boxed{x_{\max} = 2v_0\sqrt{\frac{3m}{5k}}}.$$

The speed of the ball, when the compression of the spring is $x_1$:

$$\frac{6mv_0{}^2}{5} = \frac{kx_1{}^2}{2} + \frac{6mv_1{}^2}{5}, \quad \boxed{v_1 = \sqrt{v_0{}^2 - \frac{5kx_1{}^2}{12m}}}.$$

Let check the dimensions:

$$[x_{\max}] = \frac{m}{s}\cdot\sqrt{\frac{kg}{\frac{N}{m}}} = \frac{m}{s}\cdot\sqrt{\frac{kg\cdot m}{kg\cdot\frac{m}{s^2}}} = m, \quad [v_1] = \sqrt{\left(\frac{m}{s}\right)^2 - \frac{\frac{N}{m}\cdot m^2}{kg}} = \sqrt{\frac{m^2}{s^2} - \frac{\left(kg\cdot\frac{m}{s^2}\right)\cdot m}{kg}} = \frac{m}{s}.$$

Let evaluate the quantities:

$$x_{\max} = 2\cdot 3.6\cdot\sqrt{\frac{3\cdot 1.5}{5\cdot 2}} = 4.83(m), \quad v_1 = \sqrt{3.6^2 - \frac{5\cdot 2\cdot 0.1^2}{12\cdot 1.5}} = 3.60\left(\frac{m}{s}\right).$$

**Answer**: $4.83 m$, $3.60\frac{m}{s}$.