Answer on Question#40141, Physics, Astronomy | Astrophysics

a) The comet Encke has an aphelion distance of 6.1X10^11 m and perihelion distance of 5.1X10^11 m. The mass of the sun is 2.0X10^30 kg. Find the speed of the comet at the perihelion and the aphelion.

Solution:

a) Given:

M = 2.0×10^{30} kg

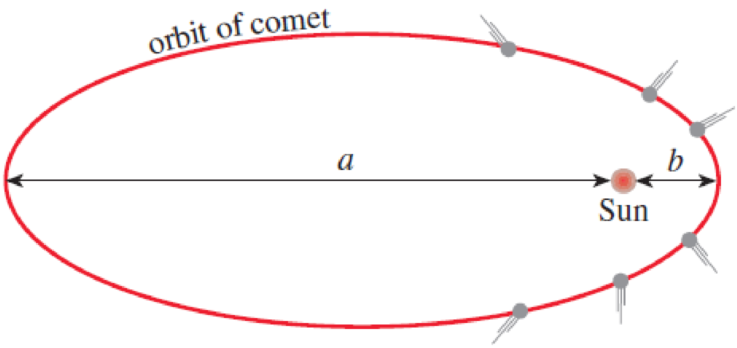
a = aphelion = 6.1×10^{11} m

b = perihelion = 5.1×10^{11} m

v_a = ?, v_b = ?

The total mechanical energy (ME) of a comet, or any orbiting body, is the sum of its kinetic energy (KE) and its gravitational potential energy (PE):

ME = KE + PE = constant



a = distance at aphelion
b = distance at perihelion

$$\mathrm{ME} = \frac{mv^2}{2} - m\frac{GM}{r} = constant$$

where M is the mass of the Sun, m is the mass of the comet, r is its instantaneous distance from the Sun and G (6.673×10^{-11} Nm^2 kg^{-2}) is the universal gravitational constant.

When PE > KE the comet will have an elliptical orbit with its total mechanical energy given by

$$\mathrm{ME} = -m\frac{GM}{2s}$$

$$-m\frac{GM}{2s} = \frac{mv^2}{2} - m\frac{GM}{r}$$

The comet has a velocity given by

$$v = \sqrt{\frac{GM}{s}\left(\frac{2s}{r} - 1\right)}$$

where s is the mean radius of its orbit (sometimes referred to as the semi-major axis).

The aphelion + perihelion = the major axis.

$$s = \frac{a+b}{2} = 5.6 \times 10^{11}\ \mathrm{m}$$

The speed of the comet at the aphelion

$$v_a = \sqrt{\frac{GM}{s}\left(\frac{2s}{a} - 1\right)} = \sqrt{\frac{6.673 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{5.6 \cdot 10^{11}}\left(\frac{2 \cdot 5.6}{6.1} - 1\right)} = 14115.7\,\frac{\mathrm{m}}{\mathrm{s}} = 14.1\ \mathrm{km/s}$$

The speed of the comet at the perihelion

$$v_b = \sqrt{\frac{GM}{s}\left(\frac{2s}{b} - 1\right)} = \sqrt{\frac{6.673 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{5.6 \cdot 10^{11}}\left(\frac{2 \cdot 5.6}{5.1} - 1\right)} = 16883.5\,\frac{\mathrm{m}}{\mathrm{s}} = 16.9\ \mathrm{km/s}$$

b) The planet Earth is 1.5X10^11 m from the sun and orbits the sun in one year. The planet Pluto takes 248 years to orbit the sun. How far is Pluto from the sun?

Solution:

The third Kepler law captures the relationship between the distance of planets from the Sun, and their orbital periods.

"The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit."

Mathematically, the law says that the expression P^2/a^3 has the same value for all the planets in the solar system, where P is period and a is distance from the Sun.

$$\frac{P_{Earth}^2}{a_{Earth}^3} = \frac{P_{Pluto}^2}{a_{Pluto}^3}$$

$$a_{Pluto} = a_{Earth}\sqrt[3]{\frac{P_{Pluto}^2}{P_{Earth}^2}}$$

$$a_{Pluto} = 1.5 \cdot 10^{11}\sqrt[3]{\frac{248^2}{1^2}} = 1.5 \cdot 10^{11} \cdot 39.473 = 59.21 \cdot 10^{11}\ \mathrm{m}$$

Answer. a) $v_a = 14.1$ km/s,) $v_b = 16.9$ km/s;

b) 59.21×10^{11} m.