Answer on Question #40139, Physics, Mechanics | Kinematics | Dynamics

a) An L-shaped object of uniform density is hung over a nail so that it is free to rotate. Determine the angle that the long side of the object makes with the vertical. The long side of the L-shaped object is given to be twice as long as the short side.

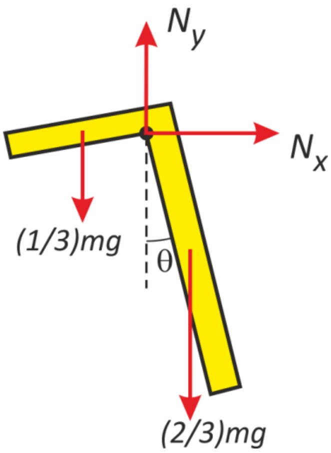
b) A solid cylinder rolls up an inclined plane without slipping. If the incline makes an angle of 30° to the horizontal and the coefficient of static friction is $\mu_s = 0.40$, find its acceleration. Also determine the angle of the inclined plane at which the object will start to slip.

Solution:

a)

We solve Static Equilibrium problems by sketching the extended free-body diagram, an FBD where the locations of the all forces are indicated so that torques can be calculated. Then we determine the three equations necessary for static equilibrium, $\Sigma Fx = 0$, $\Sigma Fy = 0$, and $\Sigma \tau z = 0$.

The forces that we know are working on the L-shaped object are a normal from the nail and the weight which acts from the center of mass. Ordinarily, for complex shapes, we first determine the CM. However, in this case, it is easier to consider the two arms of the objects as being separate objects. The long arm will have a mass (2/3)mg and the short arm will be (1/3)mg.



We consider two forces one vertical and one horizontal.

$\Sigma Fx = 0$	$\Sigma Fy = 0$
$Nx = 0$	$Ny - (1/3)mg - (2/3)mg = 0$

We will take the nail as the pivot point since this eliminates the torques from the nail.

For the torques:

r	F	angle	direction	$\tau_z = rF\sin\theta$
0	Nx	-	-	0
0	Ny	-	-	0
L/2	(1/3)mg	$\pi+\theta$	CW	$-mgL\sin(\pi+\theta)/6$
L	(2/3)mg	θ	CCW	$2mgL\sin\theta/3$

Since $\Sigma\tau_z = 0$, the equation we get is

$$-mgL\sin(\pi+\theta)/6 + 2mgL\sin\theta/3 = 0.$$

Eliminating common terms and noting $\sin(\pi+\theta) = \cos\theta$, this becomes

$$-\cos\theta/2 + 2\sin\theta = 0,$$

or

$$\sin\theta/\cos\theta = 1/4.$$

Using the identity, $\tan\theta = \sin\theta/\cos\theta$, we thus have $\theta = \arctan(1/4) = 14.0°$. The long side makes a 14.0° angle with the vertical.

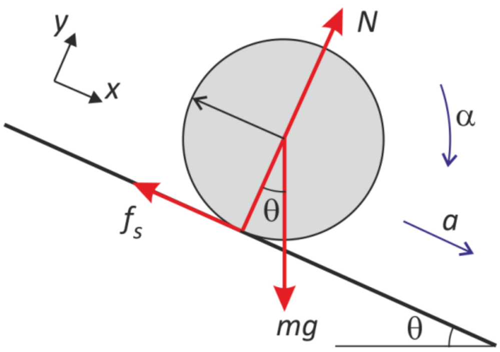
b)

Solution:

In particular for any rotating body we must draw an extended FBD in order to calculate the torques. Since the ball does not have a fixed axle, we need consider the torques acting about the center of mass (CM). Once the diagrams are drawn, we use $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$, and $\Sigma\tau_z = I_z\alpha_z$ to get a set of equations.

The cylinder is said to roll without slipping. That phrase means that the angular acceleration of the ball about its CM is related to its linear acceleration by $a = R\alpha$.

Moment of inertia $I_{cyl} = \frac{1}{2}MR^2$



$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$	$\Sigma\tau_z = I_z\alpha_z$
$mg\sin\theta - f_s = ma$	$N - mg\cos\theta = 0$	$-Rf_s = -I_{cyl}\alpha_{cm}$

The first equation is $mg\sin\theta - ma = fs$. The third equation can be simplified by using the given value for I_{cyl} and by noting that $\alpha_{cm} = a/R$. Then the third equation becomes

$fs = \frac{1}{2}ma$.

This can be substituted into the first equation to get

$mg\sin\theta - ma = \frac{1}{2}ma$.

Solving for acceleration a yields

$$a = \frac{2}{3}g\sin\theta = \frac{2}{3}\cdot 9.8\cdot\sin 30^{\circ} = 3.267\ \mathrm{m/s^2}$$

Now we will find the angle at which static friction is at its maximum, at just above this angle the object will start to slip.

We can use this result with $fs = \frac{1}{2}ma$, to get an expression for fs,

$$fs = (1/3)mg\sin\theta . \qquad (1)$$

However, the maximum value of fs is $\mu_s N$. The second equation gives $N = mg\cos\theta$, so

$$f_s = \mu_s mg\cos\theta . \qquad (2)$$

Using (1) and (2) to eliminate fs, yields

$$(1/3)mg\sin\theta = \mu_s mg\cos\theta\ .$$

Using the identity $\tan\theta = \sin\theta/\cos\theta$, we get

$$\tan\theta = 3\mu_s$$

$$\theta = \arctan(3\mu_s) = \arctan(3 \cdot 0.4) = 0.876\ \mathrm{rad} = 50.19°$$

This is the angle at which the cylinder would start to slip as it moved down the incline.

Answer. a) The long side makes a 14.0° angle with the vertical.

b) $a = 3.27\ \mathrm{m/s^2}$, $\theta = 50.19°$.