Answer on Question #40110 – Physics – Mechanics | Kinematics | Dynamics

Question: write the equation of motion of a simple harmonic oscillator which has amplitude of 5 cm and it executes 150 oscillations in 5 minutes with an initial phase of 45°. Also obtain the value of its maximum velocity.

Solution: the equation of motion of a simple harmonic oscillator is

$$\ddot{x} + \omega^2 \cdot x = 0.$$

In our case the angular frequency ω is $\omega = \frac{2\pi}{T} = \frac{2\pi}{5 \cdot 60/150} = \pi \ \frac{\text{rad}}{\text{s}}$. Finally, we obtain the equation of motion:

$$\ddot{x} + \pi^2 \cdot x = 0.$$

The solution for x is

$$x(t) = A \cdot \cos(\omega t + \phi_0),$$

Where A is the amplitude of the harmonic oscillator, $\omega -$ angular frequency, $\phi_0 -$ initial phase. In our case solution takes the form

$$x(t) = 5 \cdot \cos\left(\pi t + \frac{\pi}{4}\right).$$

From last expression we get velocity:

$$v = \dot{x}(t) = -5\pi \cdot \sin\left(\pi t + \frac{\pi}{4}\right)$$

And its maximal value is achieved in time t when $\sin\left(\pi t + \frac{\pi}{4}\right) = -1$:

$$v_{max} = 5\pi \ \frac{\text{cm}}{\text{s}}.$$

Answer:

a) The equation of motion is

$$\ddot{x} + \pi^2 \cdot x = 0.$$

b) The maximal value of oscillator's velocity is

$$v_{max} = 5\pi \ \frac{\text{cm}}{\text{s}}.$$