Answer on Question #40068, Physics, Mechanics | Kinematics | Dynamics

Question:

At a point along the line joining the centers of the earth and moon, the gravitational forces they exert on a third body are equal in magnitude but in opposite directions. Determine the distance of this point from the center of the moon, in terms of 'x', You may take the mass of the earth to be 81 times the mass of the moon, and the distance between the centers of the earth and moon to be 'x'.

Answer:

Newton's Law of Universal Gravitation states that a gravitational force between any two masses equals:

$$F = G\frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are the two masses, G is the gravitational constant, and r is the distance between the two masses.

If forces equal in magnitude:

$$F_E = F_M$$

where:

$$F_E = G\frac{M_E m}{r_E^2}$$

$$F_M = G\frac{M_M m}{r_M^2}$$

where r_E, r_M are distances to Earth and Moon, M_E, M_M are masses of Earth and Moon

Therefore:

$$G\frac{M_E m}{r_E^2} = G\frac{M_M m}{r_M^2}$$

$$\frac{M_E}{r_E^2} = \frac{M_M}{r_M^2}$$

$$r_E = r_M \sqrt{\frac{M_E}{M_M}}$$

The distance between the centers of the earth and moon equals:

$$x = r_E + r_M = r_M + r_M \sqrt{\frac{M_E}{M_M}}$$

$$r_M = \frac{x}{1 + \sqrt{\frac{M_E}{M_M}}} = \frac{x}{1 + 9} = \frac{x}{10}$$

Answer: $\frac{x}{10}$