

A child pushes his friend in a wagon along a horizontal road and then let's go. The wagon rolls for 10.0 seconds before stopping. It travels a distance of 20.0cm during the last 1.00s of its motion.

Assuming uniform acceleration. A) How fast was the wagon travelling at the instant the child released it?

B) HOW FAST WAS THE WAGON TRAVELLING WHEN IT HAD COVERED HALF OF THE TOTAL DISTANCE

Solution

A)

Assuming uniform deceleration we know

$$v_f = v_i - at = 0 \rightarrow v_i = at,$$

where v_i – initial velocity of a wagon, $v_f = 0$ – final velocity of a wagon, a - deceleration of a wagon, $t = 10.0 \text{ s}$ – total time of travel.

During the last $\Delta t = 1.00\text{s}$ of its motion a wagon travels a distance of $\Delta s = 20.0 \text{ cm}$:

$$\Delta s = \Delta v \cdot \Delta t - \frac{a\Delta t^2}{2},$$

where Δv - velocity of a wagon at time $t - \Delta t$.

$$\Delta v = v_i - a(t - \Delta t) = (v_i - at) + a\Delta t = 0 + a\Delta t = a\Delta t.$$

Let's put it in equation for distance:

$$\Delta s = a\Delta t \cdot \Delta t - \frac{a\Delta t^2}{2} = \frac{a\Delta t^2}{2}.$$

We can find uniform deceleration from it:

$$a = \frac{2\Delta s}{\Delta t^2}.$$

An initial velocity of a wagon:

$$v_i = at = \frac{2\Delta s}{\Delta t^2} t = \frac{2 \cdot 20.0\text{cm}}{1\text{s}^2} \cdot 10.0 \text{ s} = 400 \frac{\text{cm}}{\text{s}} = 4.00 \frac{\text{m}}{\text{s}}.$$

B)

v_{half} - velocity of a wagon when it had covered half of the distance.

Let's find total distance:

$$s = v_i t - \frac{at^2}{2} = v_i t - \frac{1}{2} \left(\frac{v_i}{t} \right) t^2 = \frac{1}{2} v_i t.$$

A half of the distance is

$$\frac{s}{2} = \frac{1}{4} v_i t.$$

Let's use kinematic equation

$$v_{half}^2 = v_i^2 - 2a\left(\frac{s}{2}\right) = v_i^2 - as.$$

So

$$v_{half} = \sqrt{v_i^2 - as} = \sqrt{v_i^2 - \left(\frac{v_i}{t}\right) \cdot \frac{1}{2} v_i t} = \frac{v_i}{\sqrt{2}} = \frac{4.00 \text{ m}}{\sqrt{2} \text{ s}} = 2.83 \frac{\text{m}}{\text{s}}.$$

Answer: A) $4.00 \frac{\text{m}}{\text{s}}$; B) $2.83 \frac{\text{m}}{\text{s}}$.