Answer on Question #39928 – Physics - Mechanics | Kinametics | Dynamics

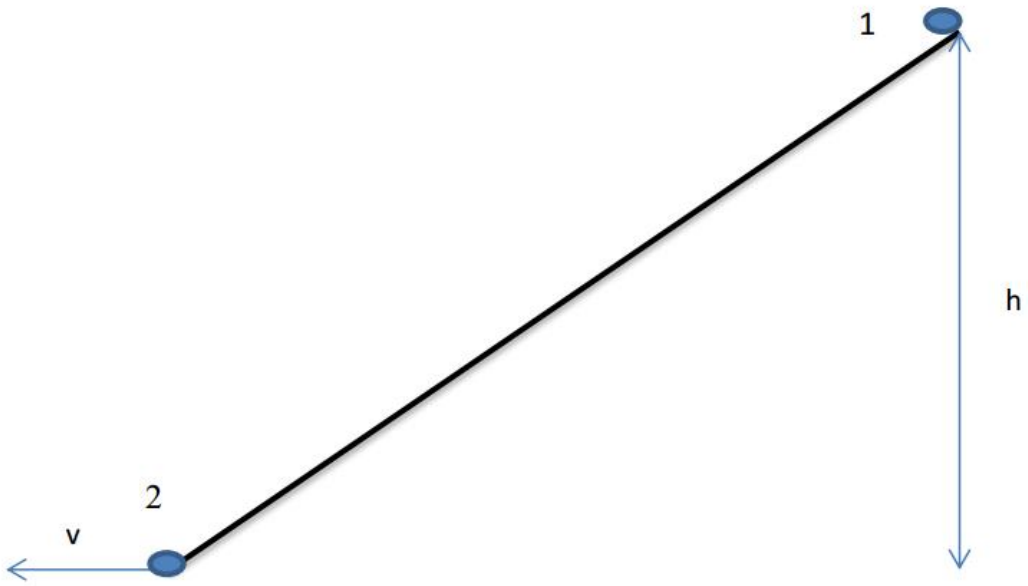
Suppose you are designing a cart for carrying goods downhill. To maximize cart speed should you design wheels so that their moment of inertia about their rotation axes are large or small or it does not matter? Explain in detail assuming that mechanical energy is conserved.

The conservation law of mechanical energy:

$$T + U = const$$

or:

$$T_1 + U_1 = T_2 + U_2$$



For state (1):

$$T_1 = 0\,, U_1 = mgh$$

For state (2):

$$U_2 = 0\,, T_2 = \frac{mv^2}{2} + \frac{Iw^2}{2}$$

$$v - cart's peed$$

$$I - sum\ of\ moments\ of\ inertia\ all\ wheels$$

$$\omega - angular\ velocity\ of\ wheels$$

$$\omega = \frac{v}{r}\,, where\ r - radius\ of\ wheel$$

Therefore:

$$mgh = \frac{mv^2}{2} + \frac{I\left(\frac{v}{r}\right)^2}{2} = \frac{mv^2}{2}\left(1 + \frac{I}{mr^2}\right)$$

$$v = \sqrt{\frac{2gh}{\left(1 + \frac{I}{mr^2}\right)}}$$

To maximize the cart speed, you should design the wheels so that their moments od inertia about their rotation axes are small as it possible. In ideal case $I \ll mr^2$ or $\frac{I}{mr^2} \ll 1$, so we can ignore $\frac{I}{mr^2}$, because it's small compare with 1 . That's mean:

$$v_{max} = \sqrt{2gh}$$

Answer:

$$v_{max} = \sqrt{2gh}$$