Answer on Question#39781, Physics, Molecular physics

One mole of oxygen at STP is adiabatically compressed to 5 atm. Calculate the final temperature. Also, calculate the work done on the gas. Take g = 1.4 and R = 8.31 J mol^-1 K^-1.

Solution

The equation of adiabatic process for an ideal gas is

$$PV^{\gamma} = const,$$

where P is pressure, V is volume, and $\gamma = 1.4$ - the adiabatic index of gas.

Also we know the state equation for an ideal gas:

$$PV = \upsilon RT,$$

where υ is the amount of substance of gas, T is the temperature of the gas and R is the universal gas constant.

So

$$V = \frac{\upsilon RT}{P} \; and \; P\left(\frac{\upsilon RT}{P}\right)^{\gamma} = const \rightarrow P^{1-\gamma} \cdot T^{\gamma} = const.$$

Hence

$$P_1^{1-\gamma} \cdot T_1^{\gamma} = P_2^{1-\gamma} \cdot T_2^{\gamma} \rightarrow T_2 = \left(\frac{P_1^{1-\gamma} \cdot T_1^{\gamma}}{P_2^{1-\gamma}}\right)^{\frac{1}{\gamma}}.$$

STP is $(P_1 = 100\ kPa, T_1 = 273\ K)$, $P_2 = 507\ kPa$.

$$T_2 = \left(\frac{(100 \cdot 10^3)^{1-1.4} \cdot 273^{1.4}}{(507 \cdot 10^3)^{1-1.4}}\right)^{\frac{1}{1.4}} = 434\ K.$$

The work done on the gas in adiabatic process is

$$W = \frac{\alpha}{2} \upsilon RT_1\left(\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right),$$

where $\alpha = \frac{2}{\gamma-1} = 5$.

$$W = \frac{5}{2} \cdot 1 \cdot 8.31 \cdot 273\left(\left(\frac{507 \cdot 10^3}{100 \cdot 10^3}\right)^{\frac{1.4-1}{1.4}} - 1\right) = 331\ J.$$

Answer: $\mathbf{434\ K; 331\ J}$.