a) 1kg of water at 0oC (degree Celsius) is fully converted into steam at 100°C at normal pressure. Calculate the change in entropy. The specific heat capacity of water is 4.18X10^3 JKg^-1K^-1 and latent heat of vaporization is 2.24x10^6 JKg^-1.

Solution

Entropy is the reversible enthalpy change of a process divided by T. Assuming the heat changes in the given situations to be reversible in nature:

$$\Delta S = \Delta S_{heating} + \Delta S_{vaporizing} = \int \left(\frac{dq_{rev}}{T}\right) + \frac{q_2}{T_{steam}}.$$

For heating up of water,

$$q = m \cdot c \cdot dT,$$

where c is the specific heat of water.

Thus,

$$\Delta S_{heating} = \int mc\left(\frac{dT}{T}\right) = mc \ln\frac{T_2}{T_1}.$$

$$q_2 = m \cdot L\,,$$

with L the latent heat of vaporization.

Therefore,

$$\Delta S = mc\ln\frac{T_2}{T_1} + \frac{m \cdot L}{T_2} = 1\text{kg} \cdot 4.18 \cdot 10^3\,\frac{\text{J}}{\text{kg K}} \cdot \ln\left(\frac{373}{273}\right) + \frac{1\text{kg} \cdot 2.24 \cdot 10^6\,\frac{\text{J}}{\text{kg}}}{373\text{K}} = 7.31\,\frac{\text{kJ}}{\text{K}}.$$

Answer: $\mathbf{7.31\,\frac{kJ}{K}}$.

b) Calculate the work done by one mole of a van der Waals' gas if during its isothermal expansion its volume increases from 1m^3 to 2m^3 at a temperature 300K. Take a = 1.39X10^-6 atm m^6 mol^-2 and b = 39.1X10^-6 m^3 mol^-1.

Solution

The work done by one mole of gas is

$$W = \int_{v_1}^{v_2} p\,dv,$$

where v is the molar volume.

The Van der Waals equation of state for 1 mole of gas is:

$$\left(p + \frac{a}{v^2}\right) \cdot (v - b) = RT.$$

From this equation we can obtain that:

$$p = \frac{RT}{v-b} - \frac{a}{v^2}.$$

The work done by one mole of a van der Waals' gas is

$$W = \int_{v_1}^{v_2} \left(\frac{RT}{v-b} - \frac{a}{v^2}\right) dv = \int_{v_1}^{v_2} \left(\frac{RT}{v-b}\right) dv - \int_{v_1}^{v_2} \left(\frac{a}{v^2}\right) dv = RT \ln \frac{v_2 - b}{v_1 - b} + a\left(\frac{1}{v_2} - \frac{1}{v_1}\right).$$

We can obtain a in SI units:

$$a = 1.39 \cdot 10^{-6} \cdot 1.01325 \cdot 10^{5} \cdot \mathrm{N} \cdot \mathrm{m}^{-2} \cdot \mathrm{m}^{6} \cdot \ \mathrm{mol}^{-2} = 0.141\ \mathrm{N} \cdot \mathrm{m}^{4} \cdot \ \mathrm{mol}^{-2}.$$

So

$$W = 8.31 \cdot 300 \cdot \ln \frac{2 - 39.1 \cdot 10^{-6}}{1 - 39.1 \cdot 10^{-6}} + 0.141 \left(\frac{1}{2} - \frac{1}{1}\right) = 1.73\ k\,J.$$

Answer: $\mathbf{1.73}\ \boldsymbol{k\,J}$.