

Answer on Question#39725, Physics, Mechanics

The linear mass density of a rod of length 1 meter is $P=(4+X)$, then the position of centre of mass from $X=0$ end of the rod is?

Solution:

The center of mass of a continuous object is located at

$$\vec{r}_{CM} = \frac{\int \vec{r} dm}{\int dm}$$

The denominator is the total mass of the rod.

$$\int dm = M$$

Since the rod is thin and lies on the x axis, $r = x$, where the sign of x would indicate the direction. The linear mass density is the mass per unit length

$$P = (4 + x) = \frac{dm}{dx}$$
$$dm = P dx = (4 + x) dx$$

Thus

$$\int dm = M = \int_0^1 (4 + x) dx = \left(4x + \frac{x^2}{2} \right) \Big|_0^1 = 4 + \frac{1}{2} = 4.5$$
$$\int r dm = \int_0^1 x(4 + x) dx = \int_0^1 (4x + x^2) dx = \left(\frac{4x^2}{2} + \frac{x^3}{3} \right) \Big|_0^1 = 2 + \frac{1}{3} = 2.33$$

The center of mass is at

$$x_{CM} = \frac{2.33}{4.5} = 0.518 \approx 0.52 \text{ m}$$

Answer. 0.52 m.