

## Answer on Question#39665, Physics, Astronomy | Astrophysics

a) The comet Encke has an aphelion distance of  $6.1 \times 10^{11}$  m and perihelion distance of  $5.1 \times 10^{11}$  m. The mass of the sun is  $2.0 \times 10^{30}$  kg. Find the speed of the comet at the perihelion and the aphelion.

### Solution:

a) Given:

$$M = 2.0 \times 10^{30} \text{ kg}$$

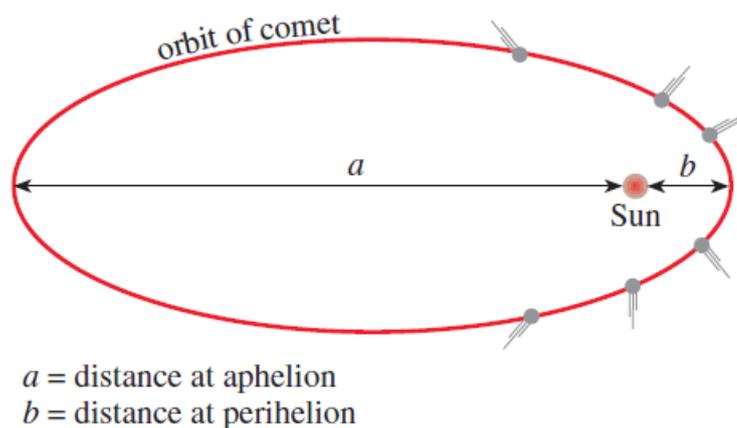
$$a = \text{aphelion} = 6.1 \times 10^{11} \text{ m}$$

$$b = \text{perihelion} = 5.1 \times 10^{11} \text{ m}$$

$$v_a = ?, v_b = ?$$

The total mechanical energy (ME) of a comet, or any orbiting body, is the sum of its kinetic energy (KE) and its gravitational potential energy (PE):

$$ME = KE + PE = \text{constant}$$



$$ME = \frac{mv^2}{2} - m \frac{GM}{r} = \text{constant}$$

where  $M$  is the mass of the Sun,  $m$  is the mass of the comet,  $r$  is its instantaneous distance from the Sun and  $G$  ( $6.673 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ ) is the universal gravitational constant.

When  $PE > KE$  the comet will have an elliptical orbit with its total mechanical energy given by

$$ME = -m \frac{GM}{2s}$$

$$-m \frac{GM}{2s} = \frac{mv^2}{2} - m \frac{GM}{r}$$

The comet has a velocity given by

$$v = \sqrt{\frac{GM}{s} \left( \frac{2s}{r} - 1 \right)}$$

where  $s$  is the mean radius of its orbit (sometimes referred to as the semi-major axis).

The aphelion + perihelion = the major axis.

$$s = \frac{a + b}{2} = 5.6 \times 10^{11} \text{ m}$$

The speed of the comet at the aphelion

$$v_a = \sqrt{\frac{GM}{s} \left( \frac{2s}{a} - 1 \right)} = \sqrt{\frac{6.673 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{5.6 \cdot 10^{11}} \left( \frac{2 \cdot 5.6}{6.1} - 1 \right)} = 14115.7 \frac{\text{m}}{\text{s}} = 14.1 \text{ km/s}$$

The speed of the comet at the perihelion

$$v_b = \sqrt{\frac{GM}{s} \left( \frac{2s}{b} - 1 \right)} = \sqrt{\frac{6.673 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{5.6 \cdot 10^{11}} \left( \frac{2 \cdot 5.6}{5.1} - 1 \right)} = 16883.5 \frac{\text{m}}{\text{s}} = 16.9 \text{ km/s}$$

b) The planet Earth is  $1.5 \times 10^{11}$  m from the sun and orbits the sun in one year. The planet Pluto takes 248 years to orbit the sun. How far is Pluto from the sun?

**Solution:**

The third Kepler law captures the relationship between the distance of planets from the Sun, and their orbital periods.

"The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit."

Mathematically, the law says that the expression  $P^2/a^3$  has the same value for all the planets in the solar system, where P is period and a is distance from the Sun.

$$\frac{P_{Earth}^2}{a_{Earth}^3} = \frac{P_{Pluto}^2}{a_{Pluto}^3}$$

$$a_{Pluto} = a_{Earth} \sqrt[3]{\frac{P_{Pluto}^2}{P_{Earth}^2}}$$

$$a_{Pluto} = 1.5 \cdot 10^{11} \sqrt[3]{\frac{248^2}{1^2}} = 1.5 \cdot 10^{11} \cdot 39.473 = 59.21 \cdot 10^{11} \text{ m}$$

**Answer.** a)  $v_a = 14.1 \text{ km/s}$ , )  $v_b = 16.9 \text{ km/s}$ ;

b)  $59.21 \times 10^{11} \text{ m}$ .