Answer on Question#39664, Physics, Mechanics | Kinematics | Dynamics

a) A particle of mass 4.0 kg, initially moving with a velocity of 5.0m/s collides elastically with a particle of mass 6.0 kg, initially moving with a velocity of -8.0m/s. What are the velocities of the two particles before and after the collision in the center of mass frame of reference? What are the velocities of the two particles after the collision in the laboratory frame?

Solution:

The velocity of the center of mass does not change by the collision:
The velocity of the system center of mass:

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{4 \cdot 5 - 6 \cdot 8}{4 + 6} = -2.8 \text{ m/s}$$

Switch to the center of mass reference frame. To do this, simply subtract v_{cm} from each particle's velocity.

$$v_{1cm} = v_1 - v_{cm} = 5 + 2.8 = 7.8 \text{ m/s}$$
$$v_{2cm} = v_2 - v_{cm} = -8 + 2.8 = -5.2 \text{ m/s}$$

Have the collision. The particles' velocities reverse.

$$v_{1cm\_after} = -7.8 \text{ m/s}$$
$$v_{2cm\_after} = 5.2 \text{ m/s}$$

Switch back to the laboratory frame of reference, by adding v_{cm} to each particle's velocity.

$$u_1 = v_{1cm\_after} + v_{cm} = -7.8 - 2.8 = -10.6 \text{ m/s}$$
$$u_2 = v_{2cm\_after} + v_{cm} = 5.2 - 2.8 = 2.4 \text{ m/s}$$

b) A 30.0 kg girl stands at the rim of a merry-go-round that has a moment of inertia of 500kgm^2 and a radius of 3.00 m. The merry-go-round is initially at rest. The woman then starts walking around the rim clockwise at a constant speed of 2.0m/s.

i) In what direction and with what angular speed does the merry-go-round rotate?

ii) How much work does the girl do to set herself and the merry-go-round into motion?

Solution:

The girls's angular velocity

$$\omega_0 = \frac{v}{R} = \frac{2.0\ m/s}{3.0\ m} = 0.67 \text{ rad/s}$$

and her moment of inertia

$$I_{girl} = mR^2 = 30 \cdot 3^2 = 270 \text{ kg m}^2$$
$$I = I_0 + I_{girl} = 500 + 270 = 770 \text{ kg m}^2$$

(i) Now

$$\omega = \frac{I_{girl}\omega_0}{I} = \frac{270 \cdot 0.67}{770} = 0.235 \text{ rad/s}$$

(ii)

Work is equal to kinetic energy of system

So

$$KE = \frac{I\omega^2}{2} = \frac{770 \cdot 0.235^2}{2} = 21.26 \text{ J}$$

Answer. a) The velocities of the two particles before and after the collision in the center of mass frame of reference:

$$v_{1cm} = 7.8 \text{ m/s}$$
$$v_{2cm} = -5.2 \text{ m/s}$$
$$v_{1cm\_after} = -7.8 \text{ m/s}$$
$$v_{2cm\_after} = 5.2 \text{ m/s}$$

The velocities of the two particles after the collision in the laboratory frame:

$$u_1 = -10.6 \text{ m/s}$$
$$u_2 = 2.4 \text{ m/s}$$

b)

(i) Direction is counterclockwise, angular speed $\omega = 0.235 \text{ rad/s}$

(ii) Work = 21.26 J.