Answer on Question#39469 – Physics - Other

Find the satellite's orbital period.

Solution:

Consider a satellite with mass m orbiting a central body with a mass of mass M. The central body could be a planet, the sun or some other large mass capable of causing sufficient acceleration on a less massive nearby object. If the satellite moves in circular motion, then the net centripetal force acting upon this orbiting satellite is given by the relationship

$$\mathrm{F_{centr}} = \frac{\mathrm{mv^2}}{\mathrm{r}} \qquad (1)$$

This net centripetal force is the result of the gravitational force that attracts the satellite towards the central body and can be represented as

$$\mathrm{F_{grav}} = \mathrm{G}\frac{\mathrm{m \cdot M}}{\mathrm{r^2}} \qquad (2)$$

$$(1) = (2)$$

$$\frac{\mathrm{mv^2}}{\mathrm{r}} = \mathrm{G}\frac{\mathrm{m \cdot M}}{\mathrm{r^2}}$$

$$\mathrm{mv^2 r} = \mathrm{GmM}$$

$$\mathrm{v} = \sqrt{\frac{\mathrm{GM}}{\mathrm{r}}}$$

Now we can find the orbital period:

$$\mathrm{T} = \frac{2\pi \mathrm{r}}{\mathrm{v}} = \frac{2\pi \mathrm{r}}{\sqrt{\frac{\mathrm{GM}}{\mathrm{r}}}} = 2\pi\sqrt{\frac{\mathrm{r^3}}{\mathrm{GM}}},$$

where r is the distance from the center of the Earth (Earth's radius + altitude), G is the gravitational constant, and M is the mass of the Earth.

Answer: orbital period $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{r^3}}{\mathrm{GM}}}$.