Show that 90 J of work is needed to increase the speed of a 20-kg cart by 3 m/s.

Solution:

In order to accomplish work on an object there must be a force exerted on the object and it must move in the direction of the force.

Work = Force x distance moved in direction of force

$$W = F \cdot d$$

Newton's Second Law

$$F = ma$$

Kinematics equation

$$2ad = v^2 - v_0^2$$

where a is acceleration, d is distance, v_0 is initial velocity and v is final velocity.

Thus,

$$W = F \cdot d = mad = \frac{m}{2}(v^2 - v_0^2)$$

When $v_0 = 0$, $v - v_0 = \Delta v = 3$ m/s

$$v^2 - v_0^2 = (v + v_0)(v - v_0) = \Delta v \cdot \Delta v = (\Delta v)^2$$

Thus,

$$W = \frac{m}{2}(v^2 - v_0^2) = \frac{m(\Delta v)^2}{2} = \frac{20 \cdot (3)^2}{2} = 90 \text{ J}$$