Answer on Question #39386, Physics, Other

Two waves 1 and 2 are present on a string:
y1 = (35 mm) Sin[(8.4m^-1)x - (15.7s^-1)t]
y2 = (35 mm) Sin[(8.4m^-1)x +- (15.7s^-1)t]

(i) Write the expression for the resultant wave, y = y1 + y2 in the form of wave function for a standing wave. (ii) Determine the x coordinates of the first two antinodes, starting at the origin and progressing towards + x direction. (iii) Determine the x coordinate of the node that is between the antinodes of part (ii).

Solution:

(i) Standing waves or stationary waves are formed by the superposition of two homogenous waves, one advancing to the right and the other advancing to the left. These two harmonic waves can be represented as

$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(kx + \omega t)$$

We have:

$$y_1 = 35\sin(8.4x - 15.7t)$$
$$y_2 = 35\sin(8.4x + 15.7t)$$
$$A = 35\text{ mm},\ k = 8.4\text{ m}^{-1},\ \omega = 15.7\text{ s}^{-1}.$$

The sum of these two waves is:

$$y = y_1 + y_2 = 35\sin(8.4x - 15.7t) + 35\sin(8.4x + 15.7t) =$$
$$= 35(\sin(8.4x - 15.7t) + \sin(8.4x + 15.7t))$$

We use the identity

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$$

If $a = kx + \omega t$ and $a = kx - \omega t$ we get:

$$\frac{a+b}{2} = kx \quad \text{and} \quad \frac{a-b}{2} = \omega t$$

Then $y = 2A\cos\omega t\sin kx$

In our case:

$$y = 2\cdot 35\cdot\cos 15.7t\sin 8.4x\text{=70}\cos 15.7t\sin 8.4x$$

(ii) The positioning of the nodes and antinodes in a standing wave pattern can be explained by focusing on the interference of the two waves. The nodes are produced at locations where destructive interference occurs.

The maximum amplitude of an element of the medium has a minimum value of zero when x satisfies the condition sin(*kx*) = 0, that is, when

kx = π, 2π, 3π, ...

Because k =2π/λ, these values for kx give

$$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots, \frac{n\lambda}{2} \qquad n = 0, 1, 2, \dots$$

Antinodes, on the other hand, are produced at locations where constructive interference occurs. The element with the greatest possible displacement from equilibrium has amplitude of 2A, and we define this as the amplitude of the standing wave. The antinodes are located at positions for which the coordinate x satisfies the condition sin kx = ± 1, that is, when

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$$

Thus, the positions of the antinodes are given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \dots, \frac{n\lambda}{4} \qquad n = 1, 3, 5, \dots$$

With k =2π/λ = 8.4 rad/m, we see that the wavelength is λ =2π/8.4 = 0.748 m. Therefore, we find that the antinodes are located at

$$x_1 = \frac{\lambda}{4} = 0.187 \text{ m}$$

$$x_2 = 3\frac{\lambda}{4} = 0.187 \cdot 3 = 0.561 \text{ m}$$

(iii) The node is located at

$$x = \frac{\lambda}{2} = \frac{0.748}{2} = 0.374 \text{ m}$$

Answer. (i) $y = 70 \cos 15.7t \sin 8.4x$

(ii) $x_1 = 0.187$ m, $x_2 = 0.561$ m

(iii) $x = 0.374$ m