

Answer on Question #39385, Physics, Other

Question:

1. A harmonic wave on a rope is described by the expression

$$y(x,t) = (4.3 \text{ mm}) \sin \left[(2\pi/0.82 \text{ m})(x + (12 \text{ m/s})t) \right]$$

What are the wave's wavelength, period, wave number, frequency, and direction of propagation.

2. For the wave in qus 1 above, determine the displacement and acceleration of the element of the rope located at $x = 0.58 \text{ m}$ at the instant $t = 0.41 \text{ s}$.

Answer:

1. Traveling sinusoidal wave is represented mathematically in terms of its velocity v (in the x direction) and wave number k as:

$$y(x, t) = A \sin(k(x - vt))$$

In our case equation of a wave is:

$$y(x, t) = 4.3 \text{ mm} \sin \left[\frac{2\pi}{0.82 \text{ m}} \left(x + \left(12 \frac{\text{m}}{\text{s}} \right) t \right) \right]$$

sign "+" means wave moving to left (opposite axis direction)

Therefore, wave number k equals:

$$k = \frac{2\pi}{0.82 \text{ m}}$$

Wavelength λ equals:

$$\lambda = \frac{2\pi}{k} = 0.82 \text{ m}$$

Period equals:

$$T = \frac{0.82 \text{ m}}{12 \frac{\text{m}}{\text{s}}} = 0.068 \text{ s}$$

Frequency equals:

$$f = \frac{1}{T} = \frac{12 \frac{m}{s}}{0.82m} = 14.63 \frac{1}{s}$$

2. Displacement at $x = 0.58m$ and $t = 0.41s$ equals:

$$y(x, t) = 4.3mm \sin \left[\frac{2\pi}{0.82m} \left(0.58m + \left(12 \frac{m}{s} \right) 0.41s \right) \right] = -4.15 mm$$

Acceleration equals:

$$a = \frac{d^2}{dt^2} (y(t)) = -4.3mm \left(\frac{2\pi * 12 \frac{m}{s}}{0.82m} \right)^2 \sin \left[\frac{2\pi}{0.82m} \left(x + \left(12 \frac{m}{s} \right) t \right) \right]$$

Acceleration at $x = 0.58m$ and $t = 0.41s$ equals:

$$a = 382 \frac{mm}{s^2}$$