

Answer on Question #39382, Physics, Other

Question:

The displacement of a particle executing SHM is given by

$$x(t) = 6\sin(5\pi)t + 8\cos(5\pi)t$$

where x is in cm and t is in seconds. Calculate the amplitude, time period and initial phase of the SHM. Also, obtain the expression for the velocity of the particle.

Answer:

$$x(t) = 6 \sin 5\pi t + 8 \cos 5\pi t$$

$$x(t) = 10 \left(\frac{1}{10} 6 \sin 5\pi t + \frac{1}{10} 8 \cos 5\pi t \right) = 10 \left(\frac{3}{5} \sin 5\pi t + \frac{4}{5} \cos 5\pi t \right)$$
$$\frac{3}{5} = \cos \varphi \text{ and } \frac{4}{5} = \sin \varphi$$

$$\text{where } \varphi = \arctan \frac{4}{3}$$

Therefore:

$$x(t) = 10(\cos \varphi \sin 5\pi t + \sin \varphi \cos 5\pi t) = 10 \sin(5\pi t + \varphi)$$

Therefore amplitude equals:

$$A = 10 \text{ cm}$$

and initial phase equals:

$$\varphi = \arctan \frac{4}{3} \cong 53.13^\circ$$

time period equals:

$$T = \frac{2\pi}{5\pi} = 0.4 \text{ s}$$

velocity of the particle equals:

$$v = \frac{dx}{dt} = \frac{d}{dt} (10 \sin(5\pi t + \varphi)) = 50\pi \cos(5\pi t + \varphi)$$