Answer on Question #39368, Physics – Acoustics

Question:

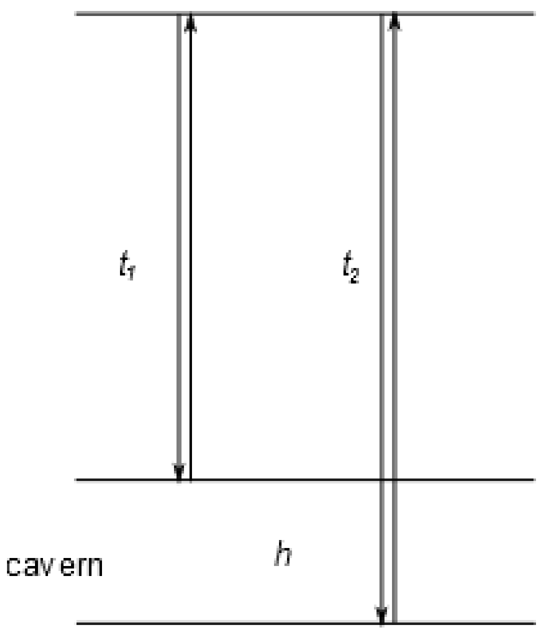
A team of geophysicists is standing on the ground. Beneath their feet, at an unknown distance, is the ceiling of a cavern. Its floor is a distance h below this ceiling. To measure h, the team places microphones on the ground. At t = 0 s, a sound pulse is sent straight downward through the ground and into the cavern. When this pulse reaches the ceiling of the cavern, one part of it is reflected back toward the microphones, and a second part continues downward, eventually to be reflected from the cavern floor. The sound reflected from the cavern ceiling reaches the microphones at t = 0.0245 s, and the sound reflected from the cavern floor arrives at t = 0.0437 s. The cavern is presumed to be filled with air at a temperature at 9 degrees Celsius. Assuming that sir behaves like an ideal gas, what is the height h of the cavern?

Answer:

$t_1 = 0.0245\ s$

$t_2 = 0.0437\ s$

$T = 9°C$



Speed of sound in the cavern is determined by the following formula:

$$c = \sqrt{\frac{\gamma R T}{M}},$$

where $\gamma = \frac{7}{5}$ is the adiabatic index for air, $R = 8.31 \frac{J}{mol \cdot K}$ is the ideal gas constant, $M = 0.029 \frac{kg}{mol}$ is the molar mass.

The time of motion of the second sound consists of the time of motion in the ground equals t_1 and of the time of motion in the cavern:

$$t_2 = t_1 + \frac{2h}{c},$$

$$h = \frac{t_2 - t_1}{2} c = \frac{t_2 - t_1}{2} \sqrt{\frac{\gamma RT}{M}}$$

$$h = \frac{0.0437 - 0.0245}{2} \sqrt{\frac{\frac{7}{5} \cdot 8.31 \cdot (273 + 9)}{29}} = 3.2\ m$$