

Answer on Question #39297, Physics – Mechanics | Kinematics | Dynamics

Question:

- 1) An object is placed on the surface of a smooth inclined plane of inclination θ . It takes time t to reach the bottom. If the same object is allowed to slide down an inclined plane of inclination θ , it takes time nt to reach the bottom where n is a number greater than 1. What is the coefficient?
- 2) The force required to just move a body up the inclined plane is double the force required to just prevent the body from sliding down the plane. The coefficient of friction is μ . Find the inclination θ of the plane.
- 3) A particle inside a hollow sphere of radius r , having coefficient of friction $(1/\sqrt{3})$, up to what height can the particle be at rest?

Answer:

- 1) The first question is confusing, so we assume that objects are placed on the same height h and the second inclined plane has the inclination θ_1 . Let us consider the first case. The object has the acceleration $a = g \sin \theta$ and moves on the distance $\ell = h \tan \theta$. So,

$$\ell = \frac{at^2}{2} = \frac{g \sin \theta t^2}{2} \Rightarrow t = \sqrt{\frac{2h}{g \cos \theta}}$$

In the same way one can obtain

$$nt = \sqrt{\frac{2h}{g \cos \theta_1}}$$

Whence

$$\frac{nt}{t} = \sqrt{\frac{2h}{g \cos \theta_1}} / \sqrt{\frac{2h}{g \cos \theta}} \Rightarrow n = \sqrt{\frac{\cos \theta}{\cos \theta_1}}$$

- 2) The friction force is

$$F_f = \mu mg \cos \theta.$$

The force required to move a body up equals

$$F_1 = \mu mg \sin \theta + F_f.$$

The force required to prevent a body from sliding down equals

$$F_2 = \mu mg \sin \theta - F_f.$$

By conditions,

$$\frac{F_1}{F_2} = 2 = \frac{\mu mg \sin \theta + \mu mg \cos \theta}{\mu mg \sin \theta - \mu mg \cos \theta}$$

$$\sin \theta = 3 \cos \theta \Rightarrow \theta = \arctan 3.$$

3) The friction force equals

$$F_f = \mu mg \cos \theta,$$

where $\mu = 1/\sqrt{3}$, $\cos \theta = \frac{r-h}{r}$, h is a height needs to be found. The friction force prevents a particle to slide down under the action of the component of the gravitational force

$$F_1 = mg \sin \theta.$$

So,

$$F_f = F_1 \Rightarrow \mu mg \cos \theta = mg \sin \theta$$

$$\tan \theta = \mu \Rightarrow \theta = 30^\circ.$$

$$\cos \theta = \frac{\sqrt{3}}{2} = \frac{r-h}{r} \Rightarrow h = \frac{2-\sqrt{3}}{2} r.$$