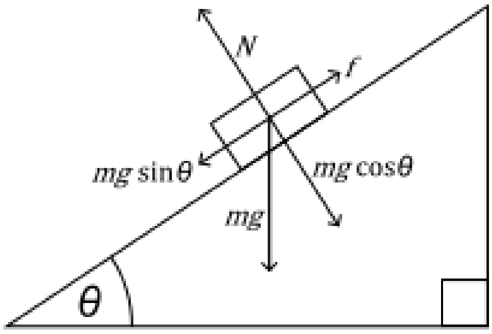
Answer on Question#39296, Physics, Mechanics | Kinematics | Dynamics

A small block starts sliding down an inclined plane forming an angle with the horizontal the coefficient of friction depends on the distance covered from rest along as ax where a=constant. find distance covered by the block down the plane, till it stops sliding, and its max velocity during this journey.

Solution:



To calculate the forces on an object placed on an inclined plane, consider the three forces acting on it.

- The normal force (N) exerted on the body by the plane due to the attraction of gravity i.e. N = mg cos θ
- the force due to gravity (mg, acting vertically downwards) F = mgsin θ
- the frictional force (f) acting parallel to the plane. f = μN = μ mg cos θ, where μ is the coefficient of friction μ =a_k x (a_k = constant).

Equation of motion:

$$ma = mg\sin\theta - f = mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - a_k\,x\cos\theta)$$

Acceleration

$$a = g(\sin\theta - a_k\,x\cos\theta)$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

$$v\frac{dv}{dx} = g(\sin\theta - a_k\,x\cos\theta)$$

$$vdv = g(\sin\theta - a_k\,x\cos\theta)dx$$

$$\int_0^v vdv = \int_0^x g(\sin\theta - a_k\,x\cos\theta)dx$$

$$\frac{v^2}{2} = gx\sin\theta - a_k g\,\frac{x^2}{2}\cos\theta$$

Block stops when velocity *v*=0 and distance x=d covered by the block down the plane

$$gd\sin\theta - a_k g\,\frac{d^2}{2}\cos\theta = 0$$

$$d = \frac{2g\sin\theta}{a_k g\cos\theta} = \frac{2}{a_k}\tan\theta$$

The max velocity we will find by taking the derivative of function and equal it to zero

$$\frac{v^2}{2} = f(x) = gx\sin\theta - a_k g\,\frac{x^2}{2}\cos\theta$$

$$f'(x) = \frac{d}{dx}\left(gx\sin\theta - a_k g\,\frac{x^2}{2}\cos\theta\right) = g(\sin\theta - a_k\,x\cos\theta) = 0$$

The max velocity will be at point

$$x = \frac{\sin\theta}{a_k\,\cos\theta} = \frac{\tan\theta}{a_k}$$

$$\frac{v_{max}^2}{2} = f\left(\frac{\tan\theta}{a_k}\right) = g\frac{\tan\theta}{a_k}\sin\theta - a_k g\,\frac{\left(\frac{\tan\theta}{a_k}\right)^2}{2}\cos\theta = \frac{g\sin\theta\tan\theta}{2a_k}$$

Thus,

$$v_{max} = \sqrt{\frac{g\sin\theta\tan\theta}{a_k}}$$

Answer. 1. $d = \frac{2}{a}\tan\theta$; 2. $v_{max} = \sqrt{\frac{g\sin\theta\tan\theta}{a}}$.