

Answer on Question #39276, Physics, Mechanics | Kinematics | Dynamics

An ambulance siren has a fundamental frequency of 261 Hz. The ambulance is headed towards an accident site with a speed of 100 km/h. Two police officers on separate motor cycles head for the same accident site: one follows the ambulance with a speed of 90 km/h and the other approaches the accident site from the other direction with a speed of 90 km/h. What frequency does the ambulance siren have for each of the police officers? Take the speed of sound equal to 340 m/s.

Solution:

When wave energy like sound waves travels from two objects, the wavelength can seem to be changed if one or both of them are moving. This is called the Doppler effect.

In classical physics, where the speeds of source and the receiver relative to the medium are lower than the velocity of waves in the medium, the relationship between observed frequency f and emitted frequency f_0 is given by:

$$f = \left(\frac{c + v_r}{c + v_s} \right) f_0$$

where

c is the velocity of waves in the medium;

v_r is the velocity of the receiver relative to the medium; positive if the receiver is moving towards the source (and negative in the other direction);

v_s is the velocity of the source relative to the medium; positive if the source is moving away from the receiver (and negative in the other direction).

The frequency is decreased if either is moving away from the other.

If the speeds v_s and v_r are small compared to the speed of the wave, the relationship between observed frequency f and emitted frequency f_0 is approximately

Observed frequency

Change in frequency

$$f = \left(1 + \frac{\Delta v}{c} \right) f_0$$

$$\Delta f = \frac{\Delta v}{c} f_0$$

where

$$\Delta f = f - f_0$$

$\Delta v = v_r - v_s$ is the velocity of the receiver relative to the source: it is positive when the source and the receiver are moving towards each other.

Given:

$$c = 340 \text{ m/s,}$$

$$f_0 = 261 \text{ Hz,}$$

$$\Delta v_1 = 90 - 100 = -10 \frac{\text{km}}{\text{h}} = \frac{-10 \times 1000}{3600} = -2.78 \text{ m/s (police officer follows the ambulance)}$$

$$\Delta v_2 = 100 - 90 = 10 \frac{\text{km}}{\text{h}} = 2.78 \text{ m/s (police officer moving towards the ambulance)}$$

$$f_1 = \left(1 + \frac{\Delta v_1}{c}\right) f_0 = \left(1 + \frac{-2.78}{340}\right) 261 = 258.87 \approx 258.9 \text{ Hz}$$

$$f_2 = \left(1 + \frac{\Delta v_2}{c}\right) f_0 = \left(1 + \frac{2.78}{340}\right) 261 = 263.13 \approx 263.1 \text{ Hz}$$

Answer. $f_1 = 258.9 \text{ Hz}$, $f_2 = 263.1 \text{ Hz}$.