

Answer on Question 39086, Physics, Optics Maxwells field equations in free space are

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

To derive wave equation we will take curl of the curl equations and use curl of the curl identity:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \cdot (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

We know that $\nabla \cdot \mathbf{E} = 0$. From the other hand, from equations we see that

$$\nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \left(\frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{\partial (\nabla \times \mathbf{B})}{\partial t} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

All together give us

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

In the very same way we get

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$