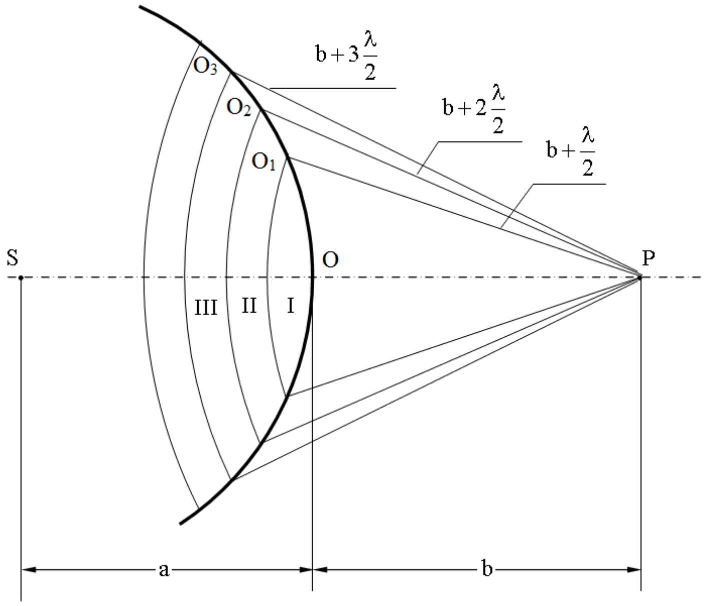
Answer on Question #39082, Physics, Optics

Show that all Fresnel half-period zones have the same area.

Solution.

Let the light source *S* extends monochromatic spherical wave, *P* - the point of observation. Through the point *O* passes spherical wave surface. It is symmetric with respect to the line *SP*.



To determine the resultant effect at *P*, Fresnel subdivided the wavefront into a number of circular zones I, II, III etc. Let *PO = b*. The radii of zones equal $b+\frac{\lambda}{2}$, $b+\frac{2\lambda}{2}$, $b+\frac{3\lambda}{2}$... etc. The area enclosed between O and O_1, O_1 and O_2, O_2 and O_3 etc. are known as half period zones. Each zone differs from its neighbour by a phase difference of π or a path difference of λ/2. The area enclosed by the first circle of radius OO_1 is called the first half period zone. The area enclosed by the annular strip O_1O_2 is known as second half period zone and so on. Thus, the annular area between (n-1)th circle and *n*th is the *n*th hall period zone.

The area of the *n*th zone:

$$S_n = \pi OO_n^2 - OO_{n-1}^2 = \pi[(PO_n^2 - PO^2) - (PO_{n-1}^2 - PO^2)] = \pi[PO_n^2 - PO_{n-1}^2] =$$
$$= \pi\left[\left(b+\frac{n\lambda}{2}\right)^2 - \left(b+\frac{(n-1)\lambda}{2}\right)^2\right] =$$
$$= \pi\left[b^2 + \frac{n^2\lambda^2}{4} + bn\lambda - b^2 - \frac{(n-1)^2\lambda^2}{4} - b\lambda(n-1)\right] =$$
$$= \pi\left[b\lambda + \frac{\lambda^2}{4}(n^2 - n^2 - 1 + 2n)\right] = \pi\left[b\lambda + \frac{\lambda^2}{4}(2n-1)\right]$$

$b \gg \lambda$ so λ^2 term is negligible.

Thus,

$$S_n \approx \pi b\lambda.$$

The area of each half period zone ($\pi b\lambda$) is approximately same and independent of the order of zone.