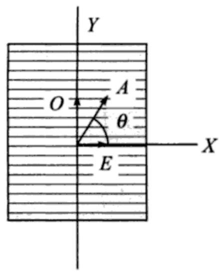
Answer on Question#39072, Physics, Optics

Show that plane polarized light and circularly polarized light are special cases of elliptically polarized light.

Solution:



Suppose that a plane polarized light beam of amplitude A is incident on a uniaxial crystal at an angle θ. Let $A\cos\theta$ and $A\sin\theta$ be the amplitudes of E-ray and O-ray respectively. If δ be the phase difference between the two emergent beams, then their vibrations can be expressed as

For E-ray: $x = A\cos\theta\sin(\omega t + \delta) = a\sin(\omega t + \delta)$ (1)

For O-ray: $y = A\sin\theta\sin\omega t = b\sin\omega t$ (2)

where $a = A\cos\theta$ and $b = A\sin\theta$

From second equation we have:

$$\frac{y}{b} = \sin\omega t$$

Hence $\cos\omega t = \sqrt{1 - \sin^2\omega t} = \sqrt{1 - \frac{y^2}{b^2}}$

From first eqation we have:

$$x = a\sin(\omega t + \delta) = a(\sin\omega t\cos\delta + \cos\omega t\sin\delta)$$

or,

$$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$$

or,

$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$$

Squaring and rearranging, we get:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta$$

This is the general equation of an ellipse.

Spesial cases:

1. When $\delta = 0$ $\sin\delta = 0$ and $\cos\delta = 1$, therefore

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos 0 = \sin^2 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\left[\frac{x}{a} - \frac{y}{b}\right]^2 = 0$$

or,

$$y = \frac{b}{a}x$$

This is the **equation of a straight line**. In this case, the emergent light is **plane polarized**.

2. When δ = π/2 sin δ = 1 and cos δ = 0, therefore

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\frac{\pi}{2} = \sin^2\frac{\pi}{2}$$

or,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

This is the **equation of an ellipse**. In this case, the emergent light is **elliptically polarized**.

When δ = π/4 sin δ = $1/\sqrt{2}$ and cos δ = $1/\sqrt{2}$, therefore

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\sqrt{2}\frac{xy}{ab} = \frac{1}{2}$$

which is again an equation of ellipse.

3. When δ = π/2 and θ = 45°, therefore a = b, and

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$$

This is the **equation of an circle.** In this case, the emergent light is **circularly polarized**.

In general, the resultant of two plane polarized beams is an elliptically polarized light. Under certain conditions (δ = 0 or, δ = π/2 and θ = 45°), however, the resultant light is plane or circularly polarized.

Thus, the plane polarized light and circularly polarized light are the special cases of elliptically polarized light.