

Answer on Question#38829, Physics, Optics

How does Fraunhofer diffraction pattern due to a single slit differ from that of a circular aperture?

Answer:

The Fraunhofer diffraction equation is used to model the diffraction of waves when the diffraction pattern is viewed at a long distance from the diffracting object.

The diffraction at a single slit of width d is shown in Figure 1. Diffraction occurs in all directions to the right of the slit.

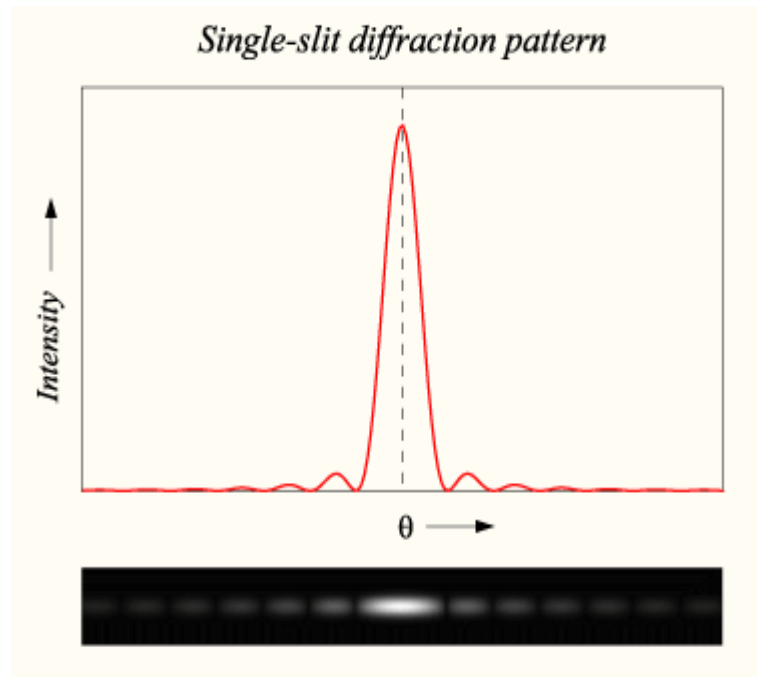


Fig.1. Graph and image of single-slit diffraction

The pattern consists of a central bright fringe (band) flanked by much weaker maxima alternating with dark fringes.

The general condition for a minimum for a single slit is:

$$m\lambda = d \sin \theta$$

where $m = 1, 2, 3, 4$ and so on

- d is the width of the slit,
- θ is the angle of incidence at which the minimum intensity occurs, and
- λ is the wavelength of the light

The intensity profile can be calculated using the Fraunhofer diffraction equation as

$$I(\theta) = I_0 \text{sinc}^2 \left(\frac{d\pi}{\lambda} \sin \theta \right)$$

where

- $I(\theta)$ is the intensity at a given angle,
- I_0 is the original intensity, and
- the unnormalized sinc function above is given by $\text{sinc}(x) = \sin(x)/x$ if $x \neq 0$, and $\text{sinc}(0) = 1$

The amplitude distribution for diffraction due to a **circular aperture** forms an intensity pattern with a bright central band surrounded by concentric circular bands of rapidly decreasing intensity (Airy pattern). The 1st maximum is roughly 1.75% of the central intensity. 84% of the light arrives within the central peak called the Airy disk.

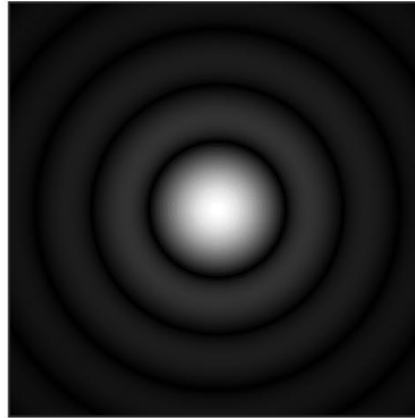


Fig.2. An image of an **Airy disk**.

Far away from the aperture, the angle at which the first minimum occurs, measured from the direction of incoming light, is given by the approximate formula:

$$\sin \theta \approx 1.22 \frac{\lambda}{d}$$

or, for small angles, simply

$$\theta \approx 1.22 \frac{\lambda}{d}$$

Where θ is in radians, λ is the wavelength of the light and d is the diameter of the aperture.

The variation in intensity with angle is given by

$$I(\theta) = I_0 \left(\frac{2J_1(k a \sin \theta)}{k a \sin \theta} \right)^2$$

where a is the radius of the circular aperture, k is equal to $2\pi/\lambda$ and J_1 is a Bessel function. The smaller the aperture, the larger the spot size at a given distance, and the greater the divergence of the diffracted beams.

Answer. The diffraction due to a **a single slit** forms a pattern with a bright central **vertical band** surrounded by vertical bands. The diffraction due to a **circular aperture** forms a pattern with a bright central **circular band** surrounded by concentric circular bands of rapidly decreasing intensity