

## Answer on Question#38829, Physics, Optics

How does Fraunhofer diffraction pattern due to a single slit differ from that of a circular aperture?

### Answer:

The Fraunhofer diffraction equation is used to model the diffraction of waves when the diffraction pattern is viewed at a long distance from the diffracting object.

The diffraction at a single slit of width  $d$  is shown in Figure 1. Diffraction occurs in all directions to the right of the slit.

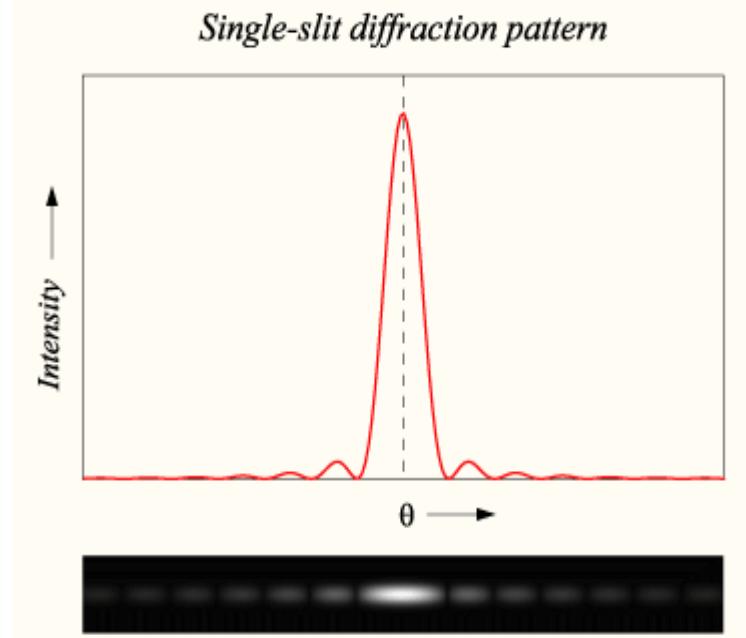


Fig.1. Graph and image of single-slit diffraction

The pattern consists of a central bright fringe (band) flanked by much weaker maxima alternating with dark fringes.

The general condition for a minimum for a single slit is:

$$m\lambda = d \sin \theta$$

where  $m = 1, 2, 3, 4$  and so on

- $d$  is the width of the slit,
- $\theta$  is the angle of incidence at which the minimum intensity occurs, and
- $\lambda$  is the wavelength of the light

The intensity profile can be calculated using the Fraunhofer diffraction equation as

$$I(\theta) = I_0 \text{sinc}^2\left(\frac{d\pi}{\lambda} \sin \theta\right)$$

where

- $I(\theta)$  is the intensity at a given angle,
- $I_0$  is the original intensity, and
- the unnormalized sinc function above is given by  $\text{sinc}(x) = \sin(x)/(x)$  if  $x \neq 0$ , and  $\text{sinc}(0) = 1$

The amplitude distribution for diffraction due to a **circular aperture** forms an intensity pattern with a bright central band surrounded by concentric circular bands of rapidly decreasing intensity (Airy pattern). The 1st maximum is roughly 1.75% of the central intensity. 84% of the light arrives within the central peak called the Airy disk.

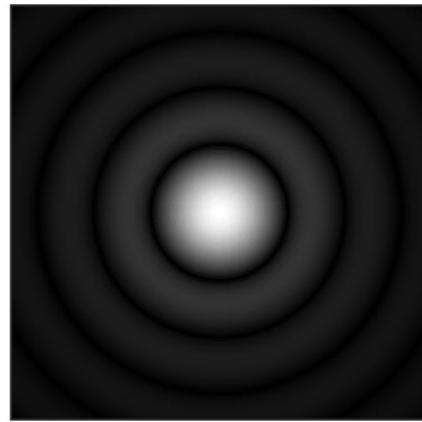


Fig.2. An image of an **Airy disk**.

Far away from the aperture, the angle at which the first minimum occurs, measured from the direction of incoming light, is given by the approximate formula:

$$\sin \theta \approx 1.22 \frac{\lambda}{d}$$

or, for small angles, simply

$$\theta \approx 1.22 \frac{\lambda}{d}$$

Where  $\theta$  is in radians,  $\lambda$  is the wavelength of the light and  $d$  is the diameter of the aperture.

The variation in intensity with angle is given by

$$I(\theta) = I_0 \left( \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right)^2$$

where  $a$  is the radius of the circular aperture,  $k$  is equal to  $2\pi/\lambda$  and  $J_1$  is a Bessel function. The smaller the aperture, the larger the spot size at a given distance, and the greater the divergence of the diffracted beams.

**Answer.** The diffraction due to a **single slit** forms a pattern with a bright central **vertical band** surrounded by vertical bands. The diffraction due to a **circular aperture** forms a pattern with a bright central **circular band** surrounded by concentric circular bands of rapidly decreasing intensity