Answer on Question#38807 - Physics, Mechanics | Kinematics | Dynamics

A Parachutist is falling with a speed of 50m/s when his parachute opens. if the air resistance is (Mv^2)/25 where M is the total mass of the man and his parachute. Find the speed of the man as a function of time t after the parachute opens. Take g=10m/s

Solution:

Taking downward to be positive, we have:

$$\frac{dv}{dt} = \frac{F}{m} = \frac{Mg - \frac{Mv^2}{25}}{M} = g - \frac{v^2}{25}$$

$$g = 10\frac{m}{s^2} \Longrightarrow$$

$$\frac{dv}{dt} = \frac{1}{25}(250 - v^2)$$

Divide both sides by $\frac{1}{25}(250 - v^2)$:

$$\frac{25\frac{dv}{dt}}{-v^2 + 250} = 1$$

Integrate both sides with respect to x:

$$\int \frac{25\frac{dv}{dt}}{-v^2 + 250} dt = \int 1\, dt$$

Evaluate the integrals:

$$-\frac{1}{2}\left(\sqrt{\frac{5}{2}}\left(\log\left(-v + 5\sqrt{10}\right) - \log\left(v + 5\sqrt{10}\right)\right)\right)$$

$$= t + c_1, \text{where } c_1 \text{ is arbitrary constant}$$

Solve for v:

$$v(t) = \frac{5\sqrt{10}\left(e^{2\sqrt{\frac{2}{5}}(t+c_1)} - 1\right)}{e^{2\sqrt{\frac{2}{5}}(t+c_1)} + 1}$$

Solve for c_1 using the initial conditions:

Substitute $v(0) = 50\frac{m}{s}$ into $v(t) = \frac{5\sqrt{10}\left(e^{2\sqrt{\frac{2}{5}}(t+c_1)} - 1\right)}{e^{2\sqrt{\frac{2}{5}}(t+c_1)} + 1}$:

$$\frac{5\sqrt{10}\left(e^{2\sqrt{\frac{2}{5}}c_1} - 1\right)}{e^{2\sqrt{\frac{2}{5}}c_1} + 1} = 50$$

Solve the equation:

$$c_1 = \sqrt{\frac{5}{2}}\log\left(\frac{1}{3}(-i)\sqrt{11+2\sqrt{10}}\right)$$

$$c_1 = \sqrt{\frac{5}{2}}\log\left(\frac{1}{3}i\sqrt{11+2\sqrt{10}}\right)$$

Substitute c_1 into $v(t) = \frac{5\sqrt{10}\left(e^{2\sqrt{\frac{2}{5}}(t+c_1)}-1\right)}{e^{2\sqrt{\frac{2}{5}}(t+c_1)}+1}$

$$v(t) = \frac{5\sqrt{10}\left(\exp\left(2\sqrt{\frac{2}{5}}\left(t+\sqrt{\frac{5}{2}}\log\left(\frac{1}{3}i\sqrt{11+2\sqrt{10}}\right)\right)\right)-1\right)}{\exp\left(2\sqrt{\frac{2}{5}}\left(t+\sqrt{\frac{5}{2}}\log\left(\frac{1}{3}i\sqrt{11+2\sqrt{10}}\right)\right)\right)+1}$$

Alternate form:

$$v(t) = -\frac{55\sqrt{10}e^{2\sqrt{\frac{2}{5}}t}}{9\left(1-\frac{1}{9}(11+2\sqrt{10})e^{2\sqrt{\frac{2}{5}}t}\right)} - \frac{100e^{2\sqrt{\frac{2}{5}}t}}{9\left(1-\frac{1}{9}(11+2\sqrt{10})e^{2\sqrt{\frac{2}{5}}t}\right)} - \frac{5\sqrt{10}}{1-\frac{1}{9}(11+2\sqrt{10})e^{2\sqrt{\frac{2}{5}}t}}$$

Approximated expanded form:

$$v(t) = 37e^{2.5t} - 19e^{1.26t} + 11e^{1.26t} - 21e^{2.5t} + \frac{1422}{173e^{1.26t}-90}$$
$$= 167 + \frac{1422}{610t-90}$$

Answer: speed of the man as a function of time t after the parachute opens:

$$v(t) = 167 + \frac{1422}{610t-90}.$$