Answer on Question #38576, Physics, Mechanics | Kinematics | Dynamics

Centre of gravity of a hollow cone?

Solution.

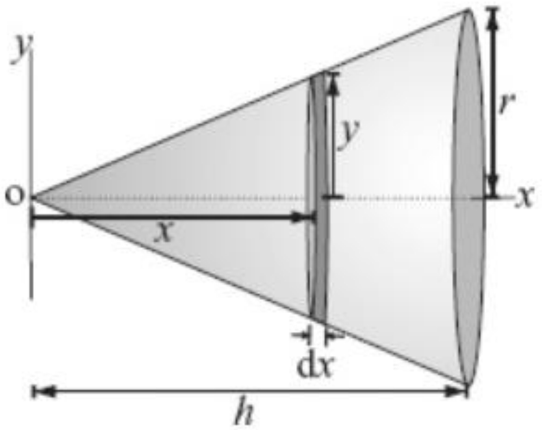
Take the apex of the cone shown in Figure as the origin. The cone is symmetric about *x*-axis, therefore y-coordinate of centre of gravity is given $y_c = 0$.

Consider the cone as split into an infinite number of rings.

I mean rings.

When we calculate center of gravity of a uniform **solid cone** we consider the cone as split into an infinite number of **disks**. When we calculate center of gravity of a **hollow cone** we consider the cone as split into an infinite number of **rings**.

Consider one such ring of thickness *dx* at x.

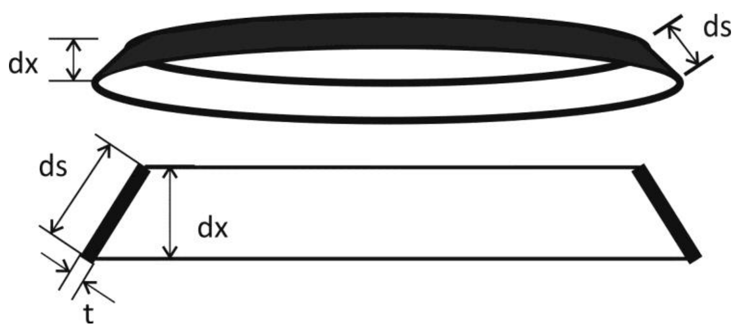


Radius of ring $y = \frac{r}{h} x$

Mass of ring $dm = \rho \cdot dV = \rho (2\pi \cdot y \cdot ds) t$

$$dm = \rho \cdot \left(2\pi \frac{r}{h\cos\alpha} x dx \right) t ,$$

where ρ – density and $\frac{dx}{ds} = \cos\alpha$, t – thickness of cone side.



ds – height of the ring's side

t – thickness of a cone side (thickness of a side of the ring).

The *x*-coordinate of centre of gravity is given by

$$x_c = \frac{\int x dm}{\int dm}$$

or

$$x_c = \frac{\int_0^h x\rho \cdot \left(2\pi \frac{r}{h\cos\alpha} x dx\right)t}{\int_0^h \rho \cdot \left(2\pi \frac{r}{h\cos\alpha} x dx\right)t} = \frac{\int_0^h x^2 dx}{\int_0^h x dx} = \frac{\left.\frac{x^3}{3}\right|_0^h}{\left.\frac{x^2}{2}\right|_0^h} = \frac{2}{3}h.$$

The distance of centre of gravity of hollow cone from the vertex is (2/3)*h* and from the base is (1/3)*h*.

Answer. $y_c = 0$, $x_c = \frac{2}{3}h$.