Answer on Question#38423 – Engineering - Other

For the free-falling parachutist with linear drag, assume a first jumper is 80 kg and has a drag coefficient of 12 kg/s. If a second jumper has a drag coefficient of 15 kg/s and a mass of 93 kg, how long will it take the second jumper to reach the same velocity as the first jumper reached in 10 s?

Solution:

Taking downward to be positive, we have:

$$\frac{dv}{dt} = \frac{F}{m} = \frac{mg - kv}{m} = g - \left(\frac{k}{m}\right)v$$

Solving the separable equation, we have

$$t + C = -m\frac{\ln\left(g - \frac{kv}{m}\right)}{k}$$

So that

$$g - \frac{kv}{m} = Ae^{-\frac{kt}{m}}, \text{where } v = 0 \text{ when } t = 0, \text{so } A = g.$$

$$v = \frac{mg\left(1 - e^{-\frac{kt}{m}}\right)}{k}$$

Therefore, after 10 seconds, the first jumper attains a velocity of

$$v = \frac{80\text{kg} \cdot 9.8\frac{N}{kg} \cdot \left(1 - e^{-\frac{12\frac{kg}{s} \cdot 10s}{80\ kg}}\right)}{12\frac{kg}{s}} = 50.8\frac{m}{s}$$

The amount of time it takes the second jumper is

$$t = -m \cdot \frac{\ln\left(1 - \frac{kv}{mg}\right)}{k} = -93\ \text{kg} \cdot \frac{\ln\left(1 - \frac{15\frac{kg}{s} \cdot 50.8\frac{m}{s}}{93\ \text{kg} \cdot 9.8\frac{N}{kg}}\right)}{15\frac{kg}{s}} = 11.2\ \text{s},$$

Answer: the amount of time it takes the second jumper is 11.2 s.