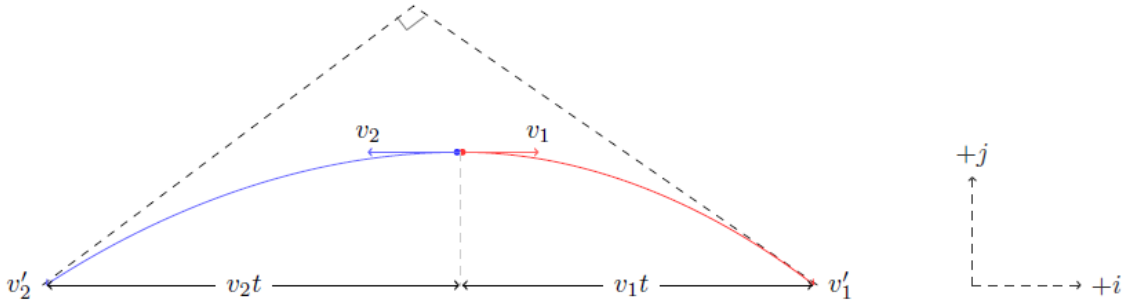
Answer on Question#38354, Physics, Mechanics

Two bodies are moving with acceleration g and velocities of 3 & 4m/s in opposite directions. What will be the distance between the two bodies when their velocity vectors become perpendicular to each other?

Solution

We can visualize the situation as given below. Suppose that the velocities of two bodies become mutually perpendicular after time t. Let the velocities of the two bodies at that instant be v_1' and v_2' , respectively. What can we say about these velocities?



• The velocity of each body can be resolved into horizontal and vertical component ($\hat{\imath}$ and $\hat{\jmath}$).

• The acceleration due to gravity g acts only on the vertical component of velocity. The horizontal component remains unchanged for both bodies.

• Since the initial velocity in the vertical direction is 0 for both bodies, they travel the same distance in vertical direction. Hence, the distance between the bodies at t is given only by the horizontal distance between the bodies at time t.

We have,

$$v_1' = v_1\hat{\imath} - gt\hat{\jmath}$$

$$v_1' = -v_2\hat{\imath} - gt\hat{\jmath}$$

At time t, the vectors v_1' and v_2' are mutually perpendicular. Hence, their dot product must be 0.

$$v_1' \cdot v_2' = 0$$

$$(v_1\hat{\imath} - gt\hat{\jmath}) \cdot (-v_2\hat{\imath} - gt\hat{\jmath}) = 0$$

$$-v_1v_2\hat{\imath}\cdot\hat{\imath} - v_{1gt\hat{\imath}}\cdot\hat{\jmath} - v_2gt\hat{\imath}\cdot\hat{\jmath} + (gt)^2\hat{\jmath}\cdot\hat{\jmath} = 0$$

$$-v_1v_2 + (gt)^2 = 0 \ [\hat{\imath}\cdot\hat{\imath} = 1 \text{ and } \hat{\imath}\cdot\hat{\jmath} = 0]$$

$$t = \frac{\sqrt{v_1v_2}}{g}$$

In time t, the bodies have moved v_1t and v_2t in opposite directions, therefore the distance between them is

$$d = (v_1 + v_2)t = (v_1 + v_2)\frac{\sqrt{v_1v_2}}{g} = (3+4)\frac{\sqrt{3\cdot 4}}{9.8} = 2.8\ m$$

Answer: $d = 2.8m$