Answer on Question#38323, Physics, Other

Two charged spheres of equal radii when kept at a certain distance attract each other by a force of F. The charges are touched with each other and placed at the same separation again. They now repel each other by a force of F/4. Find the ratio of the charges

Solution

At first the force between the charged spheres is attractive with magnitude F:

$$F = k\frac{|Q||q|}{d^2},$$

where Q – the charge with larger absolute value, q - the charge with smaller absolute value, d – the distance between charges, k – Coulomb's constant.

Since the two spheres are identical, when they are touched the charges rearrange themselves to a new equilibrium distribution which must have a like charge on each sphere. Since the total available charge is $Q + q$, each spheres has $\frac{Q+q}{2}$ of charge.

At second, the new force is repulsive and has magnitude F/4:

$$\frac{F}{4} = k\frac{\left|\frac{Q+q}{2}\right|\left|\frac{Q+q}{2}\right|}{d^2} = \frac{1}{4}k\frac{|Q+q|^2}{d^2}.$$

Now we have

$$|Q||q| = |Q+q|^2 \rightarrow |Q||q| = Q^2 + q^2 + 2Qq.$$

$Qq = -|Q||q|$ – because initially the charges has different signs. Since

$$Q^2 + q^2 + 2Qq = -Qq \rightarrow Q^2 + q^2 + 3Qq = 0.$$

We have a quadratic equation for Q.

The discriminant

$$D = (3q)^2 - 4q^2 = 5q^2.$$

Solutions are

$$Q_1 = \frac{-3q + \sqrt{5}q}{2} = q\left(\frac{\sqrt{5}-3}{2}\right) \; and$$

$$Q_2 = \frac{-3q - \sqrt{5}q}{2} = -q\left(\frac{\sqrt{5}+3}{2}\right).$$

$$Q = -q\left(\frac{\pm\sqrt{5}+3}{2}\right).$$

The ratio of the charges is

$$\frac{Q}{q} = -\left(\frac{\pm\sqrt{5}+3}{2}\right).$$

Answer: $-\left(\frac{\pm\sqrt{5}+3}{2}\right)$.