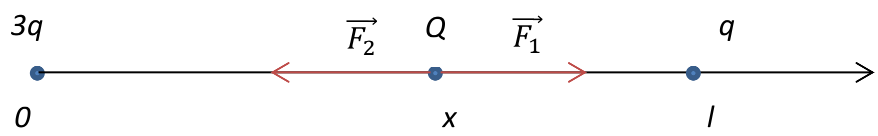
Answer on Question #38199 – Physics - Other

Question: two small beads having charges $q_1 = 3q$ and $q_2 = q$ are fixed at the opposite ends of a horizontal insulating rod of length $d = 1{,}5\ m$. The bead with charge q_1 is at the origin. A third small charged bead is free to slide on the rod.

a) At what position x is the third bead in equilibrium?
b) Can the equilibrium be stable?

Solution: let us draw the simplified picture of this situation:



The third charge is interacting with first and second ones with forces $F_1 = k\frac{3qk}{x^2}$ and $F_2 = k\frac{qk}{(l-x)^2}$, where x is the position of the charge Q. These forces have opposite directions. The total force acting on the third chard is

$$F_{tot} = k\frac{3qk}{x^2} - k\frac{qk}{(l-x)^2}.$$

In the equilibrium state $F_{tot} = 0$, so we become condition for the equilibrium position x:

$$x^2 - 3lx + \frac{3}{2}l^2 = 0 \ \rightarrow x_{1,2} = \frac{3 \pm \sqrt{3}}{2} \cdot l$$

We choose the solution for x, which is smaller than l. Finally, we obtain the equilibrium position

$$x_{eq} = \frac{3-\sqrt{3}}{2} \cdot l \cong 0{,}64 \cdot l.$$

Let us consider now small deviations from the equilibrium position x:

$$F_{tot}(x+dx) = F_{tot}(x) + F'_{tot}(x) \cdot dx = F'_{tot}(x) \cdot dx$$

Because in the equilibrium position $F_{tot}(x) = 0$. The force now takes form

$$F_{tot}(x+dx) = -2\left(k\frac{3qk}{x^3} - k\frac{qk}{(l-x)^3}\right)dx$$

In the equilibrium position $\frac{3qk}{x^2} = \frac{qk}{(l-x)^2}$ and the total force becomes after substitution

$$F_{tot}(x+dx) = -\frac{6kl}{x^3(l-x)} \cdot qQ \cdot dx$$

By the stable equilibrium the force that acts on the body by small deviations from the equilibrium state should be directed to the equilibrium position. If we pull the bead rightwards, the force should be directed leftwards (force should be negative). Because in the expression of the force $\frac{6kl}{x^3(l-x)} > 0$, qQ should be also positive. That implies that Q and q_1, q_2 should have the same sign. The same result we become when we try to pull the bead leftwards ($dx < 0$).

Answer:

a) The equilibrium position is

$$x_{eq} = \frac{3-\sqrt{3}}{2} \cdot l \cong 0,95\ m.$$

b) The equilibrium will be stable, if charges q_3 and q_1, q_2 will have the same sign.