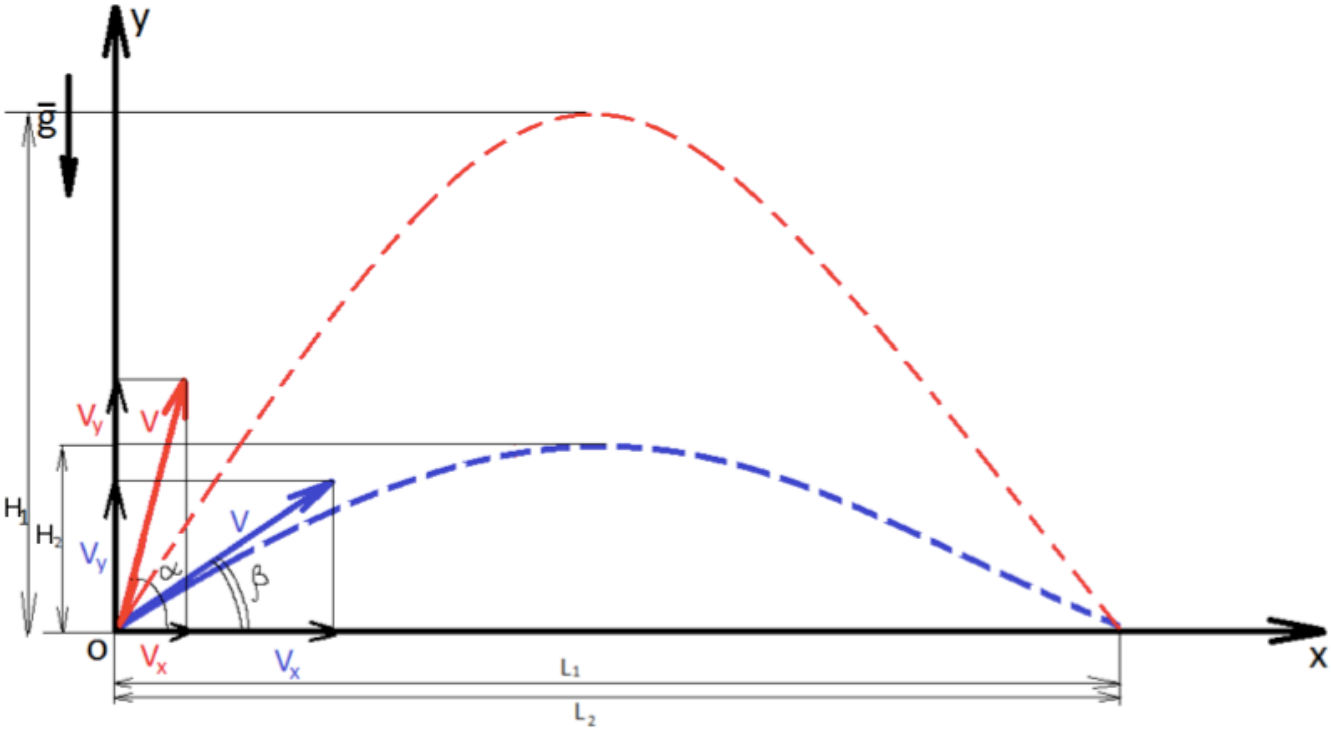
Answer on Question#38191 – Physics – Mechanics

Two particles projected from the same point with equal speeds at an angle α &β strikes the horizontal ground at the same point. If H1 & H2 be maximum height be the range for both, T1 & T2 is the time of flight. Then

1. α +β=½
2. R=4H1H2
3. T1/T2=tanα
4. Tanα=h1/h2

Solution:



We can write the equations of motion for each of the bodies:

Equations for body, thrown at angle α: (L_1 - maximum range of this body)

$$V_x = V\cos\alpha;\ V_y = V\sin\alpha;$$

$$x: L_1 = VT_1\cos\alpha\ (1), T_1 - \text{ time of the flight}$$

$$y: 0 = VT_1\sin\alpha - \frac{gT_1^{\,2}}{2}$$

$$V\sin\alpha = \frac{gT_1}{2}$$

$$T_1 = \frac{2V\sin\alpha}{g}\ (2)$$

$$(2)\text{in}\ (1): L_1 = V\frac{2V\sin\alpha}{g}\cos\alpha = \frac{2V^2\sin\alpha\cos\alpha}{g}$$

Maximum height: the time taken to reach the maximum height is equal to half of the time of flight:

$$t_1 = \frac{T_1}{2} = \frac{V\sin\alpha}{g}$$

$$\text{y: (half of the flight): } H_1 = Vt_1 \sin\alpha - \frac{g{t_1}^2}{2}$$

$$H_1 = Vt_1 \sin\alpha - \frac{g{t_1}^2}{2}$$

$$H_1 = V\frac{V\sin\alpha}{g}\sin\alpha - \frac{g\left(\frac{V\sin\alpha}{g}\right)^2}{2} = \frac{V^2\sin^2\alpha}{2g}$$

Equation for body, thrown at angle β (bodies strikes the horizontal ground at the same point):

$$L_2 = V\frac{2V\sin\beta}{g}\cos\beta = \frac{2V^2\sin\beta\cos\beta}{g} = L_1$$

$$\frac{2V^2\sin\alpha\cos\alpha}{g} = \frac{2V^2\sin\beta\cos\beta}{g}$$

$$\sin 2\alpha = \sin 2\beta$$

$$2\alpha = \pi - 2\beta$$

$$2\alpha + 2\beta = \pi$$

$$\alpha + \beta = \frac{1}{2}\pi$$

Answer: 1) $\alpha + \beta = \frac{1}{2}\pi$.