Answer on Question #38115, Physics, Other

Let φ denote the angle between vertical line and position of the pendulum. Then, from one side, torque is $\vec{M}=\vec{L}\times\vec{r}$, and $M=mgl\sin\varphi$. From the other side, according to equations of rigid body dynamics, $\frac{d\vec{L}}{dt}=\vec{M}$, and $L=I\omega$, where moment of inertia of pendulum is $I=ml^2$.

Hence $M=I\frac{d\omega}{dt}=ml^2\beta=ml^2\ddot{\varphi}$ (β is angular acceleration).

Thus, $mgl\sin\varphi=ml^2\ddot{\varphi}\Rightarrow\ddot{\varphi}-\frac{g}{l}\sin\varphi=0$. For small oscillations $\sin\varphi\approx\varphi$, hence $\ddot{\varphi}-\frac{g}{l}\varphi=0$.

General solution of this differential equation is $\varphi(t)=C\sin(\omega t-\delta)$, where $\omega=\sqrt{\frac{g}{l}}$, so motion of pendulum is harmonic. This ends the proof.

For infinite length of pendulum, $T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{l}{g}}\rightarrow\infty$, hence period is infinite (independent of length).