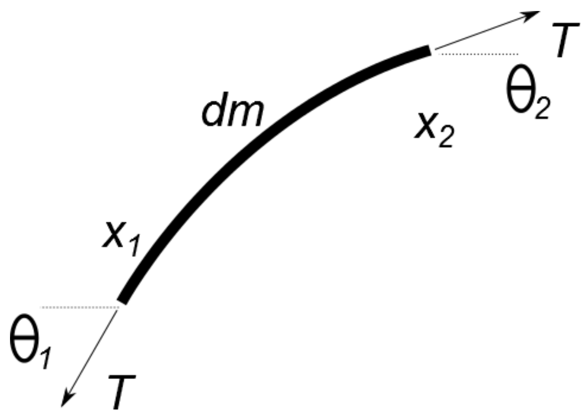
Answer on Question #38028, Physics, Mechanics

Question:

What is the role of tension in wave motion?

Answer:



Tension determines a phase speed of waves. It can be shown by the following procedure. Let us consider a string with linear density of μ. We can write a Newton's second law in the vertical direction as

$$F_y = ma_y = T\sin\theta_1 - T\sin\theta_2,$$

where m is a mass of small piece of string and T is a tension force. If one replaces each sine by derivative due to smallness of angles for the case of linear oscillations, we get the following identity:

$$\mu dx \frac{\partial^2 y}{\partial t^2} = T\left(\left.\frac{\partial y}{\partial x}\right|_{x=x_2} - \left.\frac{\partial y}{\partial x}\right|_{x=x_1}\right),$$

or

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{1}{dx}\left(\left.\frac{\partial y}{\partial x}\right|_{x=x_2} - \left.\frac{\partial y}{\partial x}\right|_{x=x_1}\right)$$

The last fraction can be replaced by the second derivative and finally we obtain the wave equation

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}.$$

Solution of this equation can be found as $y = A\sin(kx - \omega t)$. Substituting it one obtains the following relation

$$\frac{T}{\mu} = \left(\frac{\omega}{k}\right)^2$$

Because $\frac{\omega}{k} = v_{phase}$ by definition,

$$v_{phase} = \sqrt{\frac{T}{\mu}}.$$

Thus, one can see that the phase speed of a wave is determined by a restoring property (tension force T) and inertial property (mass density μ).