Answer on Question #37760 - Physics - Other

Work done in bringing a mass from infinity to center of earth.

Solution.

The mass of object is m.

We use the model: Earth is homogeneous sphere. Earth's radius is R_e, density is ρ, mass is

$$M(R_e) = \frac{4\pi}{3}\rho R_e^{\ 3}$$

We know that the gravitational potential at infinity is zero $\varphi(\infty) = 0$.

Potential inside the Earth is $\varphi(r) - \varphi(0) = \int_0^r \frac{GM(r')}{r'^2} dr' = \int_0^r \frac{G\frac{4\pi}{3}\rho r'^3}{r'^2} dr = G\frac{2\pi}{3}\rho r^2$, ***r*** is distance to Earth center, $G = 6.67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$, $M(r) = \frac{4\pi}{3}\rho r^3$ is mass of sphere, which has density ρ and radius ***r***.

If $r > R_e$ we choose $\varphi(r) = \frac{GM(R_e)}{r}$. We need the continuity of potential at $r = R_e$. Whence,

$$\varphi(R_e) = \frac{GM(R_e)}{R_e} = \frac{4\pi}{3}G\rho R_e^{\ 2} = G\frac{2\pi}{3}\rho R_e^{\ 2} + \varphi(0)$$

$$\varphi(0) = 2\pi G\rho R_e^{\ 2}$$

The work is $A = \left|m(\varphi(0) - \varphi(\infty))\right| = 2\pi Gm\rho R_e^{\ 2}$.

At surface of Earth the gravitational acceleration is $g = \frac{GM(R_e)}{R_e^{\ 2}} = \frac{4\pi}{3}G\rho R_e$.

Whence $A = \frac{3}{2}gmR_e$

Answer:

$$A = \frac{3}{2}gmR_e$$