Answer on Question#37528, Physics, Quantum Mechanics.

The Hamiltonian for quantum oscillator is $H=\frac{p^2}{2m}+\frac{mw^2x^2}{2}$. It is possible to solve Schrodinger's equation $H\psi=E\psi$ and obtain solution for each state in terms of Hermite polynomials in coordinate representation. But, if one needs only the ground state energy, it is possible to do it much more simple, without solving the general case.

Let us introduce operators $\hat{a}=\frac{1}{\sqrt{2}}\left(\sqrt{\frac{mw}{\hbar}}\hat{x}+i\frac{\hat{p}}{\sqrt{m\hbar w}}\right)$ and $\hat{a}^{+}=\frac{1}{\sqrt{2}}\left(\sqrt{\frac{mw}{\hbar}}\hat{x}-i\frac{\hat{p}}{\sqrt{m\hbar w}}\right)$.

Hamiltonian might be rewritten as $H=\hbar w\left(\hat{a}^{+}\hat{a}+\frac{1}{2}\right)$.

The commutation relations for operators $\hat{a},\hat{a}^{+}$ are $[\hat{a},\hat{a}^{+}]=1$.
Eigenfunctions for energy state are found from $a\psi_0=0$ (there are no lower levels).

Hence, $H\psi_0=\hbar w(a^{+}a+\frac{1}{2})\psi_0=\frac{\hbar w}{2}\psi_0=E_0\psi_0$, so **ground state energy is** $E_0=\frac{\hbar w}{2}$.

It is possible, looking at operator $\hat{a}=\frac{1}{\sqrt{2}}\left(\sqrt{\frac{mw}{\hbar}}\hat{x}+i\frac{\hat{p}}{\sqrt{m\hbar w}}\right)$ to find coordinate representation $\psi_0=Ce^{\frac{-mwx^2}{\hbar}}$, and using $\int|\psi_0|^2dx=1$, obtain $C=\left(\frac{mw}{\pi\hbar}\right)^{\frac{1}{4}}$, so $\psi_0=\left(\frac{mw}{\pi\hbar}\right)^{\frac{1}{4}}e^{\frac{-mwx^2}{\hbar}}$ **- this is the wave function of ground state**.