Answer on Question#37475, Physics, Mechanics

Question:

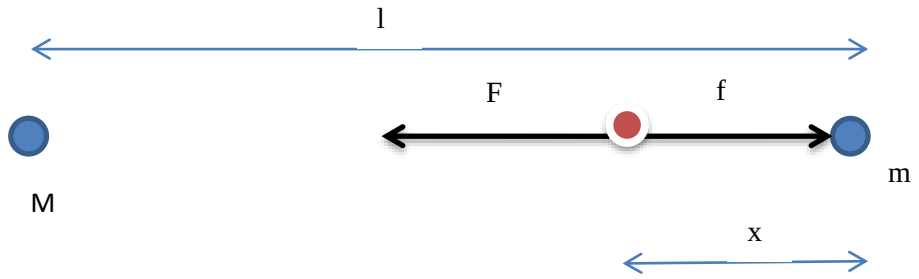
Two isolated point masses m and M are separated by a distance l. The moment of inertia of the system about an axis passing through a point where gravitational field is zero and perpendicular to the line joining the two masses, is?

Answer:

Gravitational field equals

$$g = \frac{F}{m_t}$$

where F is the gravitational force, m_t is the mass of the test particle



F, f are gravitational forces, x, l – distances

Gravitational field is zero if gravitational forces f and F are the same:

$$f = F$$

Gravitational force equals:

$$F = \frac{Gm_1m_2}{r^2}$$

Therefore:

$$\frac{m}{x^2} = \frac{M}{(l-x)^2}$$

$$\left(\frac{l-x}{x}\right)^2 = \frac{M}{m}$$

$$\frac{l}{x} - 1 = \sqrt{\frac{M}{m}}$$

Therefore x equals:

$$x = \frac{l}{1 + \sqrt{\frac{M}{m}}}$$

Moment of inertia of small body equals:

$$I = mr^2$$

where r is perpendicular distance to the specified axis

Total moment of inertia equals sum of moments of inertia:

$$I = I_m + I_M = mx^2 + M(l - x)^2 = m\left(\frac{l}{1 + \sqrt{\frac{M}{m}}}\right)^2 + M\left(l - \frac{l}{1 + \sqrt{\frac{M}{m}}}\right)^2$$

$$= \frac{ml^2}{\left(1 + \sqrt{\frac{M}{m}}\right)^2} + M\left(\frac{l\sqrt{\frac{M}{m}}}{1 + \sqrt{\frac{M}{m}}}\right)^2 = \frac{m^2 + M^2}{\left(\sqrt{m} + \sqrt{M}\right)^2} l^2$$

Answer: $\frac{m^2+M^2}{\left(\sqrt{m}+\sqrt{M}\right)^2} l^2$