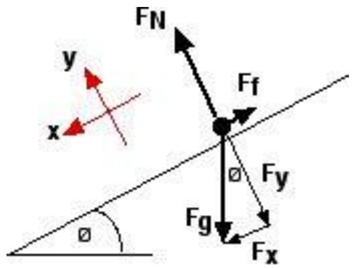
Answer on Question#37446 - Math - Algebra

A box of mass 27 kg slides down a 14° slope with a coefficient of friction of 0.16.

a) the weight of the box
b) the normal force on the box
c) the frictional force on the box
d) the total force on the box

Solution. Let us begin by making a drawing and showing the forces on the box. We also introduce a new coordinate system, so that the x axis coincides with the surface of the slope and the y axis is perpendicular to it (indicated in the drawing below with red arrows):



Here F_x is the force in the x direction, F_y is the force in the y direction, F_g is the force of gravity, F_N is the normal force and F_f is the force of friction. Finally, θ is the angle of the slope (in our case, $\theta = 14°$).

Let us now find the forces required in the question.

First, note that the ***weight*** of the box is defined as the force on the box due to gravity, and for that we have the standard formula:

$$F_g = m * g,$$

where g is the gravitational acceleration. Since the mass of the box is given ($m = 27$ kg), we take the conventional standard value for g ($g = 9.80665\ m/s^2$) and obtain

$$F_g = m * g = 27 * 9.80665 = 264.77955 \approx 264.78 \text{ (N)}.$$

We now find the x and y components of the force:

$$F_x = m * g * \sin\theta,\ \ F_y = m * g * \cos\theta.$$

Now note that the **normal force** F_N is always perpendicular to the surface of the slope, and since the box is not moving along the y axis,

$$F_N = F_y = m * g * \cos\theta.$$

Since $\cos 14° \approx 0.9703$, we have

$$F_N \approx 27 * 9.80665 * 0.9703 \approx 256.915597 \approx 256.92 \text{ (N)}.$$

We can now find the **frictional force** F_f by using the standard formula:

$$F_f = \mu * F_N,$$

where μ is the coefficient of friction (in our case, $\mu = 0.16$):

$$F_f \approx 0.16 * 256.915579 \approx 41.106496 \approx 41.11 \text{ (N)}.$$

Finally, we need to find the **total force** on the box. To do this, notice again that F_y and F_N have the opposite direction and the same value; thus, the forces cancel out each other's action. The total force on the box is then determined by F_x and F_f. Let us find F_x, remembering the formula above and using the fact that $\sin 14° \approx 0.2419$:

$$F_x = m * g * \sin\theta \approx 27 * 9.80665 * 0.2419 \approx 64.055971 \approx 64.06 \text{ (N)}.$$

Now we can find the total force on the box:

$$F = F_x - F_f \approx 64.06 - 41.11 = 22.95 \text{ (N)}.$$

Note that according to our calculations, $F_x > F_f$, and so the total force F is parallel to the surface of the slope and points downward, which is also logical — since the box should be sliding down the slope.

Answer.

a) The weight of the box is approximately 264.78 N.
b) The normal force on the box is approximately 256.92 N.
c) The frictional force on the box is approximately 41.11 N.
d) The total force on the box is approximately 22.95 N.