

A hill with a slope of 35° is situated at the top of a cliff that is 44m high. A solid disk, beginning at rest, rolls down the hill. The length of the incline is 12.55m. If a kid is standing 20m away from the cliff, will the disk hit him?

Solution:

$\alpha = 35^\circ$ – slope of the hill;

$H = 44\text{m}$ – high of the cliff;

$L = 12.55\text{m}$ – length of the incline;

$s = 20\text{m}$ – distance from the kid to the cliff;

R – radius of the disk;

S – horizontal distance that the disk traveled

Law of conservation of energy during disk is rolling down the hill:

$W_{\text{potential}} = W_{\text{kinetic}}$

$$mgL \sin \alpha = \frac{mV^2}{2} + \frac{J\omega^2}{2} \quad (1)$$

$$J \text{ – moment of inertial of the disk, } J = \frac{mR^2}{2}$$

$$\omega \text{ – angular velocity of the disk, } \omega = \frac{V}{R}$$

$$\frac{mV^2}{2} + \frac{J\omega^2}{2} = \frac{mV^2}{2} + \frac{mR^2}{4} \cdot \left(\frac{V}{R}\right)^2 = \frac{mV^2}{2} + \frac{mV^2}{4} = \frac{3mV^2}{4} \quad (2)$$

(2) in (1):

$$mgL \sin \alpha = \frac{3mV^2}{4}$$

$$4gL \sin \alpha = 3V^2$$

$$V = 2 \sqrt{\frac{gL \sin \alpha}{3}} = 2 \sqrt{\frac{9.8 \frac{\text{m}}{\text{s}^2} \cdot 12.55 \cdot \sin 35^\circ}{3}} = 9.7 \frac{\text{m}}{\text{s}} \quad (3)$$

Equations of motion for the disk after the detachment along the X-axis and Y-axis ($\beta = 90^\circ - \alpha$).

$$x: S = Vt \cos \beta = Vt \cos(90^\circ - \alpha) = Vt \sin \alpha$$

$$t = \frac{S}{V \sin \alpha} \quad (4)$$

$$y: H = Vt \sin \beta + \frac{gt^2}{2} = Vt \cos \alpha + \frac{gt^2}{2} \quad (5)$$

(4) in (5):

$$H = V \cos \alpha \cdot \frac{S}{V \sin \alpha} + \frac{g}{2} \cdot \left(\frac{S}{V \sin \alpha}\right)^2$$

$$2HV^2 \sin^2 \alpha = \frac{2S \cdot V^2 \sin^2 \alpha}{\tan \alpha} + gS^2$$

$$2HV^2 \sin^2 \alpha = 2S \cdot V^2 \sin \alpha \cdot \cos \alpha + gS^2$$

We have quadratic equation:

$$9.8 \cdot S^2 + 88.4 \cdot S - 2720 = 0$$

$$S = 12.75m$$

$12.75m \neq 20m \Rightarrow$ the disk will not hit the kid

Answer: the disk will not hit the kid (horizontal distance that traveled the disk is equal 12.75m)

