you plan a trip on which you want to average 90.0km/h. you cover the first half of your distance at an average speed of only 48km/h. what type of vehicle must you use in order to meet your goal? Note that the velocities are based on half the distance not half the time

Solution:

$V_a = 90\frac{km}{h} - \text{average speed;}$

$V_1 = 48\frac{km}{h} - \text{speed on the first half of the distance;}$

$d - \text{traveled distance;}$

Formula for the average speed:

$$V_a = \frac{d}{t} \qquad (1)$$

$$t = t_1 + t_2 = \frac{\frac{d}{2}}{V_1} + \frac{\frac{d}{2}}{V_2} = \frac{d}{2V_1} + \frac{d}{2V_2} = \frac{d(V_2 + V_1)}{2V_1V_2} \qquad (2)$$

$(2)\text{in}(1)$:

$$V_a = \frac{d}{\frac{d(V_2 + V_1)}{2V_1V_2}} = \frac{2V_1V_2}{V_2 + V_1}$$

$$2V_1V_2 = V_2V_a + V_1V_a$$

$$V_2(2V_1 - V_a) = V_1V_a$$

$$V_2 = \frac{V_1V_a}{2V_1 - V_a} = \frac{48\frac{km}{h} \cdot 90\frac{km}{h}}{2 \cdot 48\frac{km}{h} - 90\frac{km}{h}} = 720\frac{km}{h}$$

Answer: we must use the vehicle with average speed $720\frac{km}{h}$ (airplane).