A liquid of density 1.17 × 103 kg/m3 flows steadily through a pipe of varying diameter and height. At location 1 along the pipe the flow speed is 9.47 m/s and the pipe diameter is 1.11 × 101 cm. At location 2 the pipe diameter is 1.77 × 101 cm. At location 1 the pipe is 9.45 m higher than it is at location 2. Ignoring viscosity, calculate the difference between the fluid pressure at location 2 and the fluid pressure at location 1.

Solution

From the equation of continuity:

$$A_1 v_1 = A_2 v_2,$$

where $A_1 = \frac{\pi d_1^2}{4}, A_2 = \frac{\pi d_2^2}{4}$ – areas of pipes.

So

$$v_2 = \frac{A_1}{A_2}\, v_1 = \frac{\frac{\pi d_1^2}{4}}{\frac{\pi d_2^2}{4}}\, v_1 = \frac{d_1^2}{d_2^2} v_1.$$

Use Bernoulli's equation

$$P_2 + \rho g z_2 + \frac{\rho v_2^2}{2} = P_1 + \rho g z_1 + \frac{\rho v_1^2}{2}.$$

The difference between the fluid pressure at location 2 and the fluid pressure at location 1:

$$P_2 - P_1 = \rho g (z_1 - z_2) + \frac{\rho (v_1^2 - v_2^2)}{2} = \rho g (z_1 - z_2) + \frac{1}{2}\rho v_1^2 \left(1 - \left(\frac{d_1}{d_2}\right)^4\right).$$

$$P_2 - P_1 = 1.17 * 10^3 * 9.81 * 9.45 + \frac{1}{2} * 1.17 * 10^3 * 9.47^2 \left(1 - \left(\frac{1.11}{1.77}\right)^4\right).$$

$$P_2 - P_1 = 1.53 * 10^5 Pa = 153\ kPa.$$

Answer: 153 kPa.