What would be the temperature of a spherical asteroid located between mars and jupiter, twice as far from the Sun as Earth? The asteroid has no atmosphere, and its albedo is 0.15

Solution

We have that the asteroid is located on distance $R = 2a.u. = 2.992 \cdot 10^{11} m$, temperature of the sun is $T = 6000K$, its radius is $r_s = 7 \cdot 10^8 \text{m}$, radius of asteroid is r_a, albedo of asteroid is $\alpha = 0.15$, emissivity is $\varepsilon = 0.9$ (it is from observational data of asteroids), the Stefan-Boltzmann constant is $\sigma = 5.67 \cdot 10^{-8} \frac{J}{m^2 K^4 s}$

From hence, from Stefan-Boltzmann law asteroid get energy

$$Q = (1-\alpha)\frac{\pi r_a^{\,2}}{4\pi R^2} 4\pi r_s^{\,2} \sigma T^4 = (1-\alpha)\frac{\pi r_a^{\,2} r_s^{\,2} \sigma T^4}{R^2}$$

Asteroid radiates the energy $E = 4\pi r_a^{\,2} \varepsilon \sigma T_a^{\,4}$. Here T_a is temperature of asteroid.

From heat balance

$$Q = E$$

$$4\pi r_a^{\,2} \varepsilon \sigma T_a^{\,4} = (1-\alpha)\frac{\pi r_a^{\,2} r_s^{\,2} \sigma T^4}{R^2} \Rightarrow$$

$$T_a = \sqrt[4]{(1-\alpha)\frac{r_s^{\,2} T^4}{4\varepsilon R^2}} = 202K$$