The Task:

prove that photon is a massless particle.and calculate its rest mass energy.

Solution:

Let's consider a free electromagnetic field. A 4-vector potential of electromagnetic field operator is:

$$A^{\mu} = \sum_{k} \left(c_k A_k^{\mu} + c_k^{+} A_k^{\mu *} \right)$$

$$A_k^{\mu} = a_k^{\mu} e^{ikx}$$

$$A_k^{\mu *} = a_k^{\mu *} e^{-ikx}$$

Where $k^{\mu} = {p^{\mu}}/{\hbar}$ - 4-vector of momentum, $\hbar$=1-Planck's constant.

We can do a gauge transformation with 4-vector A^{μ}:

$$A^{\mu} \to A^{\mu} + \frac{\partial \theta}{\partial x_{\mu}}$$

Or

$$a_k^{\mu} \to a_k^{\mu} + \theta k^{\mu}$$

Where θ - an arbitrary function of coordinates and time. The free electromagnetic field satisfies the condition for transverse, which, if to consider the invariance of the field relative to the gauge transformation, we can write:

$$a_k^{\mu} k_{\mu} = 0$$

This condition should not be changed by gauge transformation. It means that the square of the 4-vector $k_{\mu} k^{\mu} = 0$, or ${E}/{c^2} - p^2 = 0$, or $E = pc$, thet is wright for the particle, which rest mass is equal zero.

The rest mass energy is calculated by this formula:

$$E_0 = mc^2$$

Where $m = 0$. So the energy of the rest mass of the photon is also equal zero.