Steel wire of length 2.5 m and area of cross-section $2.5 * 10^{-6} m^2$ is suspended from a torsion head. A 5 kg weight is suspended at its free-end. Calculate the work done on the wire.
Take = $2 * 10^{11} \frac{N}{m^2}$. .

Solution

Work done in the wire of length l to a length $l + \Delta l$ by force F is

$$W = \frac{1}{2} \sigma \varepsilon V,$$

where $\sigma = \frac{F}{A}$ – stress, A – area, $\varepsilon = \frac{\Delta l}{l}$ – strain, V – volume.

$$Y = \frac{\sigma}{\varepsilon} \to \varepsilon = \frac{\sigma}{Y}.$$

where Y - Young's modulus.

So

$$W = \frac{1}{2} \sigma * \frac{\sigma}{Y} * V = \frac{\sigma^2 V}{2Y}.$$

A force F in that case is weight of 5 kg:

$$F = mg,$$

where g acceleration due to the gravity.

Now we have

$$W = \frac{\left(\frac{mg}{A}\right)^2 A * l}{2Y} = \frac{m^2 * g^2 * l}{2 * A * Y}.$$

$$W = \frac{5^2 \text{ kg}^2 * 9.8^2 \left(\frac{N}{kg}\right)^2 * 2.5 \text{ m}}{2 * 2.5 * 10^{-6} m^2 * 2 * 10^{11} \frac{N}{m^2}} = 6 * 10^{-3} J.$$

Answer: $6 * 10^{-3} J$.