A gas is suddenly compressed to 10 times 3 its original pressure. Calculate the rise in temperature of the gas if its initial temperature is 27°C(γ =1.5).

Solution

Sudden compression means there is hardly any time for heat exchange to occur with the environment which indicates that the process is adiabatic for which $(P * V^{\gamma})$ is constant and γ is adiabatic index.

Also, for an ideal gas,

$$PV = \nu RT \rightarrow V = \frac{\nu RT}{P},$$

where P – pressure, V – volume, T – temperature, R - the gas constant, ν - amount of substance.

The initial value of $(P * V^{\gamma})$ is equal to its final value:

$$P_1 * {V_1}^{\gamma} = P_2 * {V_2}^{\gamma}.$$

Then

$$P_1 * \left(\frac{\nu RT_1}{P_1}\right)^{\gamma} = P_2 * \left(\frac{\nu RT_2}{P_2}\right)^{\gamma} \rightarrow \left(\frac{T_2}{T_1}\right)^{\gamma} = \left(\frac{P_2}{P_1}\right)^{\gamma-1}.$$

So

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}.$$

The rise in temperature of the gas

$$\Delta T = T_2 - T_1 = T_1 * \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - T_1 = T_1\left(\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$\Delta T = (27 + 273)K * \left((10^3)^{\frac{1.5-1}{1.5}} - 1\right) = 300K * \left((10^3)^{\frac{1}{3}} - 1\right) = 300K * (10 - 1) = 2700K.$$

Answer: $\mathbf{2700}\boldsymbol{K}$.