A one metre long rod is placed vertically and allowed to fall to the ground. if the lower end does not slip, with what velocity will the upper end hit the ground?

Solution:

$R = 1m - length\ of\ the\ rod;$

$m - mass\ of\ the\ rod.$

Law of conservation of energy:

$$W_1 = W_2 \qquad (1)$$

Initial potential energy of the rod:

$$W_1 = mg \cdot \frac{R}{2} \qquad (2)$$

Rotational energy of the rod before hitting the ground, potential energy is zero ($J -$ moment of inertia of the rod):

$$W_2 = \frac{J\omega^2}{2} \qquad (3)$$

Moment of inertia of the rod from the table of moments of inertia:

$$J = \frac{1}{3}mR^2 \qquad (4)$$

Connection between linear velocity of the upper end of the rod and the angular velocity:

$$\omega = \frac{V}{R} \qquad (5)$$

$(5)\text{and}(4)\text{in}(3)$:

$$W_2 = \frac{J\omega^2}{2} = \frac{1}{3}mR^2 \cdot \frac{1}{2} \cdot \frac{V^2}{R^2} = \frac{1}{6}mV^2 \quad (6)$$

$(6)\text{and}(2)\text{in}(1)$:

$$mg \cdot \frac{R}{2} = \frac{1}{6}mV^2$$

$$3gR = V^2$$

$$V = \sqrt{3gR} = \sqrt{3 \cdot 9.8\frac{m}{s^2} \cdot 1m} = 5.42\frac{m}{s}$$

Answer: the upper end will hit the ground with the velocity $5.42\frac{m}{s}$.